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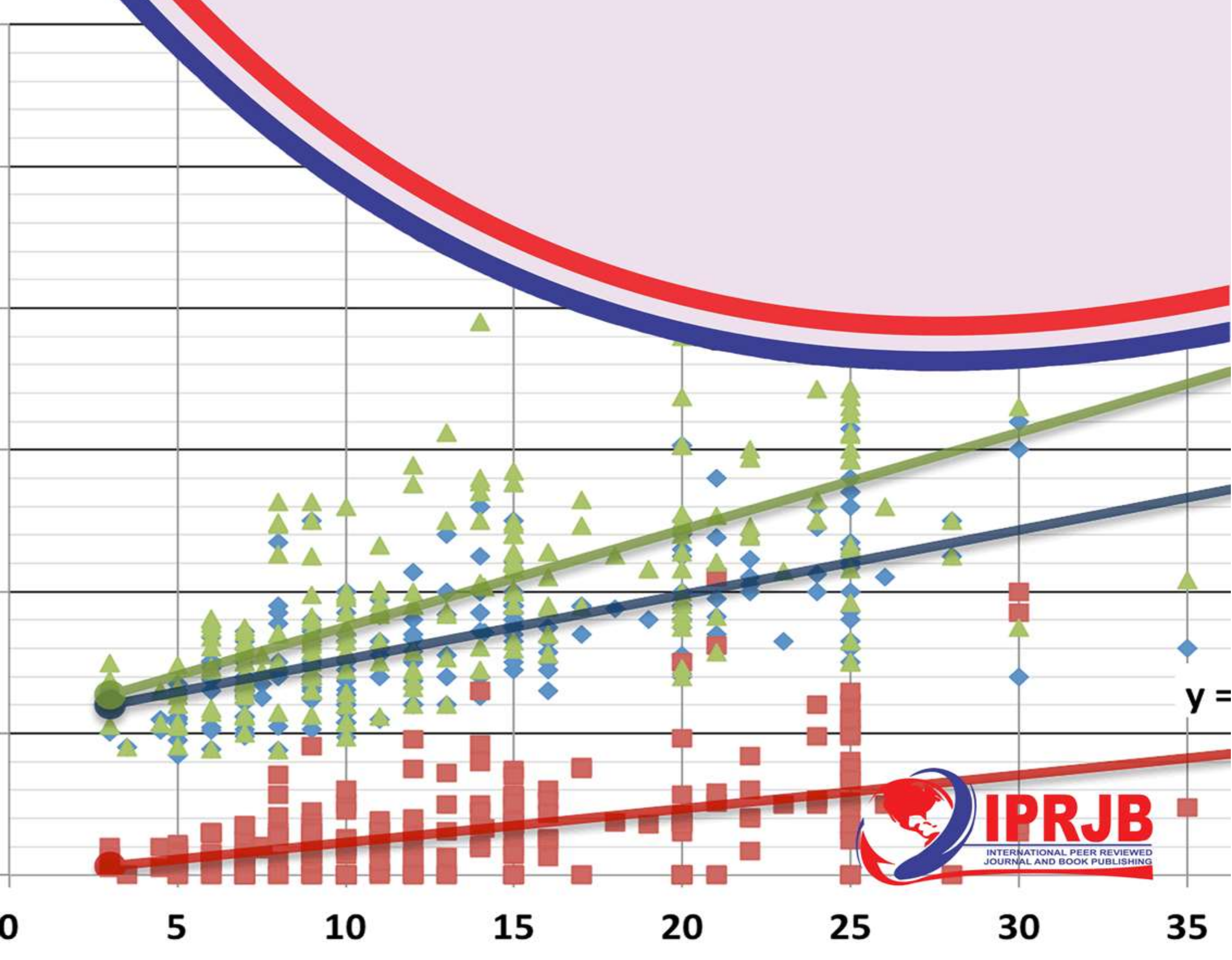


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# MODELLING ENERGY MARKET VOLATILITY USING GARCH MODELS AND ESTIMATING VALUE-AT-RISK

[1*]Simon Kinyua Weru
[1]Post Graduate Student: Jomo Kenyatta University of Agriculture and Technology
*Corresponding Author's E-mail: symohweru@gmail.com
[2]Antony Waititu
Lecturer: Jomo Kenyatta University of Agriculture and Technology
[3]Antony Ngunyi
Lecturer: Dedan Kimathi University of Technology

**Abstract**

**Purpose:** The study focused on modelling the volatility of energy markets spot prices using GARCH models and estimating Value-at-Risk.

**Methodology:** The conditional heteroscedasticity models are used to model the volatility of gasoline and crude oil energy commodities. In estimating Value at Risk; GARCH-EVT model is utilized in comparison with other conventional approaches. The accuracy of the VaR forecasts is assessed by using standard statistical back testing procedures.

**Results:** The empirical results suggests that the gasoline and crude oil prices exhibit highly stylized features such as extreme price spikes, price dependency between markets, correlation asymmetry and non-linear dependency. We also conclude that the EGARCH-EVT model is more robust, provides the best t and outperforms the other conventional models in terms of forecasting accuracy and VaR prediction. Generally, the GARCH-EVT model can be used to plays an integral role as a risk management tool in the energy industry.

**Unique contribution to theory, practice and policy:** In light of the research findings, the study recommends that organizations should leverage modern technology as a basis of realizing efficiency, effectiveness, and sustainability of projects. The study likewise recommends that organizations should build capacities to enhance labour productivity. In addition, the study recommends that organizations should adopt transformational leadership approaches as a basis of enhancing performance. The study recommends the need to revise the legal framework with a view to ensure that it reflects the changing needs of the project requirements.

**Keywords:** *Back testing, extreme value theory (EVT), Peak-over-threshold (POT), GARCH-EVT model, Value-at-Risk (VaR).*

## 1.0 INTRODUCTION

The energy industry involves the production and sale of energy including; fuel extraction, manufacturing, refining and distribution. The use of energy has been key in the development of the world. The energy industry has rapidly expanded in the recent past and become increasingly interdependent hence increasing energy consumption signifying a reliance on the energy and its related products for continued and sustainable economic growth development. Global energy supplies have become a scarce commodity which has resulted into competition for safe and affordable energy that has moved from inside the borders of national markets-partly out onto the European market. Energy resources for which sufficient supply and demand exist can be traded. These energy resources are mainly electricity, gas, coal and oil. Energy as a commodity has its characteristics such as tradability, deliverability and liquidity where this means that it has an active market with buyers and sellers constantly transacting with each other. Energy markets are commodity markets that deal specifically with the trade and supply of energy.

The main markets within energy exchange are the spot market, for short term trading and the forward market, where the physical delivery takes place at a future date. The significance of energy trading has grown rapidly in Europe and United States as a result of increased energy consumption as well as market integration. Almost no country can cover its energy needs from its own sources today. Energy trading offers the possibility to ensure the needed supply of energy is continuous and protects supply shortages and price fluctuations. The value of energy trades can change over time with market conditions and the underlying price variables. The rise of competition and deregulation in energy markets has led to relatively free energy markets that are characterized by high price shifts. For now, the leading markets of energy are China, followed by United States and then India.

Energy prices are the monetary or non-monetary costs associated with the production, distribution and consumption of the energy commodities in the energy markets. Pricing of energy commodities are subject to variety of factors that drives energy prices such as weather, political events and crises, government regulations and source fuels. Changes in one or more of these variables can intensify volatility and cause dramatic price shifts in the market. Therefore, pricing in the energy market is due to basic principles of demand and supply that are responsible for price fluctuations. Energy prices, which are largely linked to oil prices, are a major concern for the economies of the world. The price of oil refers to the spot price of a barrel (it is 42 US gallons, which is about 159 litres) of a benchmark crude oil which is a reference price for buyers and sellers of crude oil such as Organization of the Petroleum Exporting Countries (OPEC). Like any other commodity, the pricing of energy commodities especially oil will determine the overall performance in the energy markets and also act as an indicator of the performance of the global economy and prices for all other commodities. The price of oil is set at global commodity Exchanges like New York Mercantile Exchange (NYMEX). As a result, there is need to be aware of the risks involved and factors that influence the pricing strategies to be applied, and hence this will result in good decision making for investors.

With regards to modelling volatility, it is recognized in the econometrics literature that financial return series are often heteroscedasticity showing alternating volatility clusters of high and low volatility over time. The fluctuations of prices in the energy markets can be modelled using autoregressive conditional heteroscedasticity (ARCH) model introduced by Engle (1982) and the generalized conditional heteroscedasticity (GARCH) model by Bollerslev (1986) are widely established and most commonly used in modelling return variance processes in financial time series. Choosing GARCH processes to model the univariate risk factor evolution ensures by construction that the conditional variances of the univariate distributions are time-varying. Many researchers further provide evidence of volatility asymmetries, which means that negative news have a larger impact on volatility than positive news (Brandt & Kang (2004); Koutmos, 1998; Liu & Maheu, 2007). There is an extensive literature on modeling GARCH of energy prices. Many of the applications favour modifications of GARCH, which allow for the asymmetric effects, like, asymmetric GARCH and EGARCH models. Li *et al.* (2016) used data sets of Europe Brent and West Texas Intermediate (WTI) Cushing crude oil daily prices from 4th Jan, 2000 to 4th Jan, 2006. The VaR forecasting performance of GARCH-type models was analyzed and compared in a short horizon. Based on the Kupiecs POF-test and Christoffersens interval forecast test, as well as a Back-testing VaR loss function. They applied four different GARCH-VaR models with student-t distribution to forecast the conditional variance and its corresponding VaR. The Back-testing indicated that for Europe Brent crude oil, EGARCH (1, 1) model with student-t distribution had the smallest VaR loss, so it forecasts the future VaR better than other models. While for WTI crude oil, APARCH (1, 1) and GJR-GARCH (1, 1) models under student-t distribution outperformed other GARCH models. Also these results gave significant guidance on how to choose a better risk management model for certain commodity of different companies even in the same period, that is, even for same commodity (oil), even though data sets are taken in the same time interval, the commodity of a different country or companies may have a different appropriate model to predict the future Value at Risk.

## 2.0 LITERATURE REVIEW

Hasan *et al.* (2013) examined the estimates and compared the asymmetry and persistence of volatility of crude oil, natural gas and coal. The study also evaluated the effect of recent Global Financial Crisis (GFC) on the return and volatility of these energy prices. Threshold GARCH (TGARCH) and fractionally integrated GARCH (FIGARCH) models were employed. The estimated results showed that coal return volatility exhibits strong mean reversion whereas crude oil and natural gas return volatility endures shocks for relatively higher period. The estimated results also showed that volatility of crude oil and natural gas increases after positive shocks in prices.

Saltik *et al.* (2016) analysed the return volatility of spot market prices of crude oil (WTI) and natural gas (Henry Hub) for two different terms which covered 2nd Jan, 2009 to 28th Apr, 2014 and 4th Jan, 2010 to 28th Apr, 2014 with different GARCH class models such as GARCH, IGARCH, GJR-GARCH, EGARCH, FIGARCH and FIAPARCH. Their main aim of employing various GARCH models was to determine which one of these linear and non-linear asymmetric models perform more accurate. Therefore, the study was to determine a model which ensures to get a maximum return with response to the minimum loss for returns of the investments held by individual investors and fund managers, private sector budget planning decision makers, and state agencies forecasting about macroeconomic indicators. The asymmetric and Integrated GARCH models gave relatively more accurate performance than other available models. For minimum loss model, FIGARCH under skew Student-t perfomed better for first period and EGARCH under generalized error distribution was appropriate for the second period for WTI crude oil and Henry

Hub natural gas series by considering of Mean Square Error (MSE) and Mean Absolute Error (MAE) criterions.

Musaddiq (2012) modelled the volatility of crude oil futures and assessing the forecasting ability of the ARCH family of models. Historical volatility was used for modeling purposes through the ARCH family of models and made dynamic forecasts of future volatility. The study found the presence of asymmetric effects in the light, sweet and crude oil futures traded on the New York Mercantile Exchange (NYMEX). Of the ARCH models, the GJR-GARCH (1, 2) was able to make the most accurate forecasts with Threshold ARCH, TARCH (1, 1) as a close second. Therefore, when volatility forecasts for oil futures are used for hedging and pricing purposes, asymmetric rather than symmetric models are best used. Additionally, also found that trading volume and open interest are unable to reduce volatility persistence in the futures. Asymmetric power models and fractionally integrated models-also of the ARCH family could be used to analyze volatility behavior.

Fasanya and Adekoya (2017) noted in their study that good news in price changes has the tendency of increasing volatility than bad news. Also core inflation is more persistent in volatility than headline inflation. Comparing the volatility models, the symmetric models (GARCH and GARCH-M) prove to be less appropriate in modeling inflation volatility than asymmetric models (EGARCH and TGARCH). Categorically, EGARCH establishes the best fit. In connection with their findings more realistic proactive measures may be required by monetary policy authorities to promote price stability. Inflation volatility is really a serious issue for economic concerns, due to its impending uncertainty or risks concerned with economic agents. To lenders, evidence of high inflation volatility is a discouragement. It also discourages savings as people would be afraid of reduction in the real value of their saved earnings, hence, investment is jeopardized and growth is aggregately retarded.

Kang and Yoon (2013) examined the ability of three different GARCH-class models, with four innovation distributions; Gaussian (normal), Generalized Error Distribution, Student and skew Student distributions, to capture the volatility properties of natural gas futures contracts traded on the New York Mercantile Exchange. They jointly estimated the long-memory processes for conditional return and variance investigating the long-memory and persistence of long and short maturities contracts. Also examined the ability of these models and distributions forecast of the conditional variance. They found that AR(FI)MA-FIAPARCH model with a skewed Student distribution is the best model to use for short-term contracts, while the ARMA-FIAPARCH model with a Student distribution is better in defining the long-term contracts. However, there is no single innovation distribution that provides a better fit for all of the data examined. The asymmetry of shocks was also found to be significant only for the shortest contracts and is negative, meaning higher volatility for positive shocks. Further, the persistence decreases as the maturity of contracts increases. This means that the long-term investors are likely to suffer less due to price shocks, since such shocks are less persistent in the long-term.

Aduda *et al.* (2016) employed statistical techniques to investigate and model financial time series trends in energy markets. They used daily closing price for a period of about 10 years for Cushing OK WTI, RBOB and number 1 heating oil spot and futures contracts traded in the New York Mercantile Exchange (NYMEX) were considered. Also investigated the existence of stylized facts in these series in order to fit an appropriate model that adequately describes the market dynamics. They found that return series are indeed mean stationary, but are definitely not variance stationary. For the crude futures return series, the best model turned out to be an ARMA (6, 11) and for crude spot was an ARMA (7, 11) based on their Akaike Information Criterions (AICs). However, these resultant models contravened the Gaussian innovation assumption. Finally they proposed a combined ARMA (p, q)-GARCH (P, Q) model to capture the ARCH effects in the variance, which they found that the best model under these circumstances to be ARMA (0, 0)-GARCH (1, 1) implying a constant mean conditional variance equation. They concluded that GARCH models can therefore adequately model the trends and patterns in the energy markets. The trends also depict time varying variability and high persistence of oil price shocks, where these shocks therefore have a significant impact on the prices of energy prices.

Bouseba and Zeghdoudi (2015) focused their study on the profit of Generalized Auto-regressive Conditional Heteroscedasticity (GARCH) models and their applications to the Value at Risk. They presented an empirical application of a range of univariate GARCH models to oil price data for the period 01 January 2009 to 31 December 2014, a total of 2192 observations. Found that normal GARCH models explain some of the non-normality of the distribution of energy prices. When they do, the error term still exhibits skewness and leptokurtosis. To higher confidence levels, normal GARCH based estimates of energy VaR perform marginally better than the ones commonly used by energy companies. To account for non-Gaussian distribution of energy returns and changing volatility, used the stable GARCH.

Halilbegovic and Vehabovic (2016) examined that it is well-known that the usage of Value at Risk forecast is widespread. Since there is no such a method which predicts the accurate forecast, certain backtesting procedures should be undertaken in order to evaluate whether calculated VaR results are satisfactory or not. Backtesting is definitely a necessity; however, more back tests should be done to confirm the accuracy and reliability of the VaR model validation. This fact indicates that the backtesting should be a part of daily VaR calculations. The results from backtesting are able to provide information whether potential problems or risks exist in the company's core system, so in that way Company's management can take necessary risk mitigation measures and protect company against the potential future risk. The most used back-testing test is known as Kupiec POF (Proportion of failures) test.

Omari (2017) examined the performance of conventional univariate VaR models including; unconditional normal distribution model, EWMA model, Historical Simulation, Filtered Historical Simulation, GARCH normal and GARCH Students-t models in terms of forecasting accuracy. Also examined the performance of VaR models by assessing the conditional and unconditional interval coverage of the various approaches of forecasting Value at Risk. He found that GJRGARCH model and Filtered Historical Simulation models performs the best among all the VaR models based on the two back-testing measures. The GARCH-Student t models performed better than GARCH-normal models. Also, the models with Gaussian assumption generally underestimate VaR as they fail to capture the leptokurtosis which is in financial returns. HS and unconditional normal methods perform the worst for the two back-testing measures.

First, we begin by applying rolling window estimation and using univariate ARMA (1, 1) - EGARCH (1, 1) model to obtain the parameters for the most appropriate conditional mean and volatility models. Using the standardized residuals obtained from the fitted ARMA-EGARCH (1, 1) model, we then apply EVT for tail modelling to obtain the iid uniforms that are used for estimating the one-day-ahead forecasts. We capture the performances of these models fully by applying them to stock markets.

Finally, a thorough out-of-sample back-testing exercise is conducted to evaluate the performance of the conditional GARCH-EVT model, as well as a number of benchmark models, in forecasting Value-at-Risk (VaR) accurately. In order to have a conclusive analysis, we implement statistical tests, namely, the test of Kupiec (1995), the autocorrelation test of Christoffersen (1998).

## 3.0 METHODOLOGY

In this section, we specify the procedure for modelling the marginal distributions of the spot prices returns. The ARMA-GARCH-EVT approach assumes that the returns are ergodic processes (Boltzmann, 1896) and the residuals are independently identically distributed (i.i.d.) random variables.

### 3.1 ARMA-GARCH model

In the ARMA-GARCH approach, the mean equation is driven by the recursive volatility process. In line with previous studies, we assume that the conditional mean µit follows an ARMA (p, q) process and that the conditional variance σ 2 it follows a GARCH (1,1) process. The ARMAGARCH model is used to filter out serial dependence and heteroscedasticity in the returns. The ARMA model compensates for autocorrelation and the GARCH model for heteroscedasticity.

The ARMA(p, q)-GARCH(1,1) model can be modelled as:

$$r_t = \phi_0 + \sum_{i=1}^{m} \phi_i r_{t-i} - \sum_{j=1}^{n} \varphi_j \varepsilon_{t-j} + \varepsilon_t, \tag{1}$$

$$\begin{aligned} r_{it} &= \mu_{it} + \varphi r_{t-1} + \sigma_t z_t + \theta \sigma_{t-1} z_{t-1}, \\ \sigma_t^2 &= \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \end{aligned} \tag{2}$$

where rit denote the actual return for assets i = 1, 2, . . . , zit is the standardized residuals, and parameter restrictions are ω > 0, αi ≥ 0, βj ≥ 0, αi + βi < 1, ϕi + θi 6= 0. zt ∼ tν is a standardized Student-t distribution to compensate for the fat tails of the log return series and t = σtzt . Under the assumption of independently and identically distributed innovations, zt, and for f(zt ; ν) density the density function, the log-likelihood of {rt(Θ)} for a sample of T observations is given by;

$$L(\{r_t\};\Theta) = \sum_{t=1}^{T} \left[ \ln\left(f\left(z_t(\Theta);\nu\right)\right) - \frac{1}{2}\ln\left(\sigma_t^2\left(\Theta\right)\right) \right] \tag{3}$$

Where Θ is the vector of parameters that have to be estimated for the conditional mean, conditional variance and density function. Maximum likelihood estimates of the parameters are obtained by numerical maximization of the log-likelihood function using the Marquardt algorithm (Marquardt, 1963). The quasi maximum likelihood estimator (QMLE) is used since, according to Bollerslev and Wooldridge (1992), it is generally consistent, has a normal limiting distribution and provides asymptotic standard errors that are valid under non-normality. The standardized residuals Zˆ t of an ARMA (1,1)-GARCH(1,1) model are given by:

$$\hat{Z}_t = \frac{1}{\hat{\sigma}_t}\left(X_t - \hat{\mu} - \hat{\phi}X_{t-1} - \hat{\theta}\sigma_{t-1}\hat{Z}_{t-1}\right), \quad (4)$$

Where ˆσt , μ, ˆ φ, ˆ ˆθ, are the estimated parameters for σt , μ, φ, θ. Even though returns are not independent from one day to the next, the ARMA-GARCH model produces a series of iid observations that let us more closely satisfy the requirements of EVT.

### 3.2 Extreme value theory

The tail behavior of the asset returns can be modeled via extreme value theory. Extreme value theory originally introduced by Fisher and Tippett (1928) is a powerful and yet fairly robust framework which provides a comprehensive theoretical foundation and a parametric form for modeling the tail behavior of random variables. There are two main approaches of extracting extreme events from a sample of observations namely; the Block Maxima Method (BMM) which is based on modeling the distribution of a series of maxima (minima) extreme realizations, and the Peaks Over Threshold (POT) method, which models the distribution of exceedances over a given high threshold.

#### 3.2.1 Block Maxima Method

Suppose that Xt , t = 1, 2, … , n is a sequence of independently and identically distributed (i.i.d.) random variables with a common unknown distribution function FX(x) = P(Xt ≤ x), which has mean (location parameter) μ and variance (scale parameter) $\sigma^2$ . Denote the sample maxima1 of the first *n* observations by Mn = max(X1, · · ·, Xn), n ≥ 2 and let R denote the real line. The distribution function of Mn is given by:

$$Pr\{M_n \le x\} = Pr\{X_1 \le x, \cdots, X_n \le x\} = \prod_{i=1}^{n} F(x) = F^n(x) \tag{5}$$

The limit theory in univariate extremes for the block maxima, $M_n$, is given by the following theorem: *(Fisher and Tippet Fisher and Tippett (1928), Gnedenko Gnedenko (1943)):* Let $\{X_n\}$ be a sequence of i.i.d. random variables. If there exists sequences of constants $a_n > 0$, $b_n \in \mathbb{R}$ and some non-degenerate distribution function $H$ such that $(M_n - b_n)/a_n$, the cdf of normalized maxima, converges in distribution to $H(x)$, i.e.

$$P\left(\frac{M_n - b_n}{a_n} \le x\right) = F^n(a_n x + b_n) \to H(x), \quad \text{as} \quad n \to \infty, \tag{6}$$

If this condition holds then $F$ is in the maximum domain of attraction of $H$ and $F \in MDA(H)$, for some non-degenerate function then $H(x)$ belongs to one of the three standard extreme value distributions;

$$\begin{aligned} \text{Fréchet}: \ \Phi_\alpha(x) &= \begin{cases} 0, & x \le 0; \\ \exp\left(-x^{-\alpha}\right), & x > 0; \quad \alpha > 0 \end{cases} \\ Weibull: \ \Psi_\alpha(x) &= \begin{cases} \exp\left[-\left(-x^{-\alpha}\right)\right], & x \le 0, \ \alpha < 0; \\ 1, & x > 0; \end{cases} \\ Gumbel: \ \Lambda(x) &= \exp\left(-e^{(-x)}\right), x \in \mathbb{R}. \end{aligned} \tag{7}$$

The Fisher and Tippett (1928) theorem recognize that the limiting distributions of these extremes is the generalized extreme value (GEV) distribution. In this case, extreme value theory plays the same fundamental role as the Central Limit theorem plays when modelling sums of random variables. In both cases, the theory tells us what the limiting distributions are.

### 3.2.2 The Generalized Extreme Value (GEV)

Distribution According to Jenkinson (1955) the standard extreme value distributions; Frechet, Weibull and Gumbel distributions can be subsumed under a single parametrization known as the generalized extreme value distribution (GEV). The distribution function of the standard GEV is given by:

$$H_\xi(x) = \begin{cases} \exp\left\{-(1+\xi x)^{-1/\xi}\right\}, & \text{if} \quad \xi \ne 0, \\ \exp\left\{-\exp(-x)\right\}, & \text{if} \quad \xi \ne 0, \end{cases} \tag{8}$$

Where x is such that $1 + \xi x > 0$ and $\xi = 1/\alpha$ is known as the shape parameter. The shape parameter plays an important role as it characterizes the distribution of $H\xi$. If we introduce location and scale parameters $\mu$ and $\sigma > 0$ respectively, we can extend the family of distributions. The GEV $H\xi,\mu,\sigma(x)$ is defined $H\xi\ ((x - \mu)/\sigma)$ and $H\xi,\mu,\sigma$ is of type $H\xi$.

### 3.2.3 Peaks Over Threshold Method

The Peaks Over Threshold (POT) method considers the distribution of the exceedances over a certain threshold. Let Xt , t = 1, · · · , $n$ be a sequence of i.i.d. random variables with unknown distribution function F, and a certain high threshold u. The distribution function Fu is the conditional excess distribution function (McNeil *et al.*, 2015) and is given by

$$F_u(y) = P(X - u \leq y | X > u), \quad 0 \leq y \leq x_F - u \tag{9}$$

Where $y = X - u$ are the excesses over a specified threshold $u$, $x_F \leq \infty$ is the right endpoint of the distribution function $F$. The conditional excess distribution function $F_u(y)$ represents the probability that the value of $X$ exceeds the threshold $u$ by at most an amount $y$ given that $X$ exceeds the threshold $u$. This conditional probability can be written as:

$$F_u(y) = \frac{\Pr\{X - u \leq y, X > u\}}{\Pr(X > u)} = \frac{F(u+y) - F(u)}{1 - F(u)}. \tag{10}$$

Since $X = u + y$ for $X > u$, $F(x)$ has the following representation

$$F(x) = [1 - F(u)]\, F_u(y) + F(u). \tag{11}$$

For a large class of underlying distribution functions $F$, the conditional excess distribution function $F_u(y)$, for an increasing threshold u can be approximated by

$$F_u(y) \approx G_{\xi,\sigma}(y), \qquad u \to \infty,$$

where

$$G_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \frac{\xi}{\sigma} y\right)^{-1/\xi} & if \quad \xi \neq 0 \\ 1 - \exp\left(-y/\sigma\right) & if \quad \xi = 0 \end{cases} \tag{12}$$

for $y \in [0, (x_F - u)]$ if $\xi \geq 0$ and $y \in [0, -\frac{\sigma}{\xi}]$ if $\xi < 0$. $G_{\xi,\sigma}$ is the so-called Generalized Pareto Distribution (GPD).

### 3.2.4 The Generalized Pareto Distribution

The Generalized Pareto Distribution (GPD) is the limiting distribution of the peak over threshold approach. The GPD is usually expressed as a two parameter distribution with density function given by

$$G_{\xi,\sigma}(x) = \begin{cases} 1 - (1 + \xi x/\sigma)^{-1/\xi} & \text{if} \quad \xi \neq 0 \\ 1 - \exp(-x/\sigma) & \text{if} \quad \xi = 0, \end{cases} \quad (13$$

Where $\sigma > 0$, and the support is $x \geq 0$ when $\xi \geq 0$ and $0 \leq x \leq -\sigma/\xi$, when $\xi < 0$. Again the family can be extended by adding a location parameter $u$. Additionally, the shape parameter $\xi$ in the GPD exactly equals that of the corresponding GEV distribution and acts as the dominant factor in determining the tail properties of the GPD and thus measures the fatness of the tail Coles Coles et al. (2001). Similar to the GEV distribution, the GPD is generalized in the sense that it subsumes a number of other specific distributions under its parametrization. When $\xi > 0$, the distribution function $G_{\xi,\sigma}$ is the parameterized version of a heavy tailed ordinary Pareto distribution; when $\xi$

*= 0* we have a light tailed exponential distribution and when $\xi < 0$ we have a short tailed Pareto type II distribution. The ordinary Pareto distribution, where shape parameter $\xi > 0$, is most relevant in financial analysis since it is heavy tailed.

An important step and also quite challenging task in applying the POT approach is the selection of an appropriate threshold value u. The principle of threshold selection is to balance the reliability of the asymptotic approximation versus the sample variance of estimators as well. The threshold must be sufficiently high to ensure the threshold excesses have a corresponding approximate distribution within the domain of attraction of the generalized Pareto family. However, the threshold cannot be too high as this will reduce the sample information for inferences. The selection of an appropriate threshold is a compromise between bias and variance. Once a threshold level has been selected, the parameters of the GPD are estimated. In this thesis, the method of the maximum likelihood estimation (MLE) is used to estimate the shape parameter $\xi$ and the scale parameter $\sigma$ of the GPD. Given that $X_1, X_2, \cdots, X_n$ are $n$ iid sample of losses and assuming a selected threshold $u$ let $\tilde{X}_1, \tilde{X}_1, \cdots, \tilde{X}_{N_u}$ denote the exceedances. The exceedances are those $X_i$'s for which $X_i - u > 0$ and excesses over $u$ are calculated by $Y_i = \tilde{X}_i - u$, for $j = 1, \cdots, N_u$. MLE requires a sample of independently and identically distributed random variables, therefore we make an assumption that $Y_1, \cdots, Y_{N_u}$ are iid. This assumption could be criticized in practice, as in reality large losses frequently occur in clusters which implies that they are dependent. But we will assume independence of $Y_1, \cdots, Y_{N_u}$ with $Y_i \sim F_u$. For a high enough threshold $u$, and considering the Theorem (**??**), the series of excess losses can be considered to follow GPD with unknown parameters $\xi$ and $\sigma$. The log likelihood function is given by:

$$L(\xi, \sigma) = \begin{cases} -N_u \log(\sigma) - \left(\frac{1}{\xi} + 1\right) \sum_{i=1}^{N_n} \log\left(1 + \frac{\xi Y_i}{\sigma}\right), & \text{if} \quad \xi \neq 0 \\ -N_u \log(\sigma) - \frac{1}{\sigma} \sum_{i=1}^{n} Y_i, & \text{if} \quad \xi = 0 \end{cases} \tag{14}$$

By maximizing the log likelihood function subject to the constraints $\sigma > 0$ and $\left(1 + \frac{\xi Y_i}{\sigma}\right) > 0$, for all $i$, we can obtain the maximum likelihood estimates of the shape parameter $\xi$ and the scale parameter $\sigma$.

### 3.2.5 Estimation of Tail of Loss Distribution

For a sufficiently large threshold $u$, $F_u(y)$ converges to the GPD, and since $x = y + u$ for $X > u$, we have

$$F(x) = [1 - F(u)]G_{\xi,\sigma,u}(x - u) + F(u). \quad (15)$$

The function $F(u)$ is estimated non-parametrically using the empirical cdf,

$$\hat{F}(u) = \frac{1}{n}\sum_{i=1}^{n} I_{(X_i<u)} = \frac{n - N_u}{n} = 1 - \frac{N_u}{n} \quad (16)$$

where $n$ is the total number of observations and $N_u$ represents the number of exceedances beyond a given high threshold $u$. Thus, from equations (15) and (16), the distribution function $F(x)$ is given as follows

$$\hat{F}(x) = 1 + \frac{N_u}{n}\left(G_{\hat{\xi},\hat{\sigma}}(x - u) - 1\right). \quad (17)$$

Therefore, the tail estimator becomes

$$\hat{F}(x) = 1 - \frac{N_u}{n}\left(1 + \frac{\hat{\xi}}{\hat{\sigma}}(x - u)\right)^{-1/\hat{\xi}} \quad (18)$$

### 3.3 Value-at-Risk

Value-at-Risk (VaR) is the worst expected loss over a given time interval under normal market conditions of a given confidence interval. The *V aRt (α)* at the *(1-α)* quantile is defined as;

$$Pr[r_t \leq VaR_t(\alpha)] = \alpha \quad (19)$$

where $r_t$ is the return at time $t$. Equation (19) computes the probability that the returns at time horizon t will be less than or equal to $VaR_t(\alpha)$ where $\alpha$ is a percentage. The Value at Risk at time $t$ is computed as shown in the equation below

$$VaR_t = F^{-1}(\alpha)\sigma_t \quad (20)$$

Given that $F^{-1}(\alpha)$ is the corresponding quantile of the assumed distribution and σt is the forecast of the usual conditional standard deviation at time $t - 1$. This means that with probability (1-α), the potential loss encountered by the holder at financial position over the time horizon t is less than or equal to V aRt(α). Hence the negative sign signifies a loss. A measure of risk is said to be coherent if it satisfies the following properties; monotonicity, sub-additivity, positive homogeneity and translation invariance. The Value at Risk measure becomes a coherent measure of risk when it is assumed that the distribution of losses are normally distributed to satisfy sub-additivity property, otherwise it is not a coherent measure of risk. Value at Risk as a measure of risk has different methods which are used in calculating risk. VaR uses different types of methods to estimate risk including; IGARCH, Empirical quantile, Econometric modelling, traditional EVT and GARCH-EVT model based on the exceedance over a high threshold. The following methods are used in estimating value-at-Risk of spot prices of crude oil and gasoline commodities.

### 3.3.1 IGARCH Model

The IGARCH Model is equivalent to an IGARCH model with normally distributed errors. It assumes that the continuously compounded daily returns (log returns) follow a conditional normal distribution. The conditional volatility from the model is similar to an Exponentially Weighted Moving Average (EWMA) in which the weighting parameter λ is set to be 0.94 for daily data. Let rt denote the daily return and the information set available at time t − 1 by Ft−1. IGARCH assumes that $rt \mid Ft-1 \sim N(\mu t, \sigma^2_t)$. The method assumes that the conditional mean and variance evolve over time. Its structure is given as;

$$\sigma_t^2 = (1-\lambda)r_{t-1} + \lambda\sigma_{t-1}^2 \quad (21)$$

where equation (21) above is an IGARCH (1, 1) process without a drift since $\mu_t = 0$. The value of $\alpha$ is often in the interval [0.9, 1]. For the 1-step ahead volatility forecast of equation (21) is given as;

$$\hat{\sigma}_{t+1}^2 = (1-\lambda)r_t + \lambda\sigma_t^2 \quad (22)$$

Under the special IGARCH (1, 1) model, the conditional variance of $r_t[k]$ is proportional to the time horizon k. The conditional standard deviation of a k-period horizon log return is then given as $\sqrt{k}\sigma_{t+1}$. For the long financial position, loss occurs when there is a big price drop (that is, a large negative return). If the confidence level is set to be 5%, then the IGARCH uses $1.65\sigma_{t+1}$ to measure the risk of the portfolio. Therefore, under IGARCH, Value at Risk is estimated as;

$$VaR(k) = \sqrt{k} \times VaR \quad (23)$$

where,

$$VaR = \text{Amount of position} \times 1.65\sigma_{t+1} \quad (24)$$

$$VaR(k) = \text{Amount of position} \times 1.65\sqrt{K} \times \sigma_{t+1} \quad (25)$$

Equation (23) is for the k-day horizon Value at Risk of the portfolio and is known as the "square root of time rule" in VaR calculation under IGARCH. Equation (24) is the daily VaR of the portfolio under IGARCH and equation (25) is the k-day horizon where its standard deviation is in percentage. Given that this model assumes that returns follow a normal distribution, this is not typically the case for financial returns for daily frequency. This leads to an underestimation of risk. In addition, the standard assumption is that the risks associated with long positions (left quantile) and short positions (right quantile) are equal. That is, the risk is symmetrical.

### 3.3.2 Econometric modelling

GARCH models by Bollerslev (1986) have gained fast acceptance and popularity in the literature devoted to the analysis of financial time series. These time series models captures important features of the financial series, such as volatility clustering and leptokurticity. As compared to EVT-based models, GARCH models do not focus directly on the returns in the tails but instead, by recognising the tendency of financial return volatilities to be time dependent. GARCH models explicitly model the conditional volatility as a function of past conditional volatilities and returns.

In estimating Value at Risk (VaR) with GARCH type models, it is commonly supposed that the innovation distribution follows a normal distribution "conditional normal distribution" so that an estimate of VaR is given by;

$$VaR_{t+1}, q = \mu_{t+1} + \sigma_{t+1}\Phi^{-1}(q) \tag{26}$$

### 3.3.3 Historical Simulation (HS)

Historical simulation (HS) is a non-parametric method that is widely used as a method for estimating Value at Risk due to its simplicity in implementing and ease of interpretation. It involves simulating of cumulative distribution function of returns over time for estimating VaR. The HS approach to calculating VaR assumes that all past and that historically simulated distribution is identical to the returns distributed over time. HS simplifies the procedure for computing the VaR since it does not make any distributional assumption about the returns of prices. Therefore, the VaR based on HS is simply the empirical quantile of the distribution associated with the desired likelihood level. Hence VaR is given as;

$$VaR_{t+1}(q) = \text{Quantile}((r_t)_{t=1}^{n}) \tag{27}$$

Historical Simulation (HS) has a number of advantages. First, it is easy to understand and implement it. Second, its completely non-parametric and does not depend on any distribution assumption, thus capturing the non-normality in the data. The most notably disadvantage of HS is that it is impossible to obtain an out-of-sample VaR estimate with HS. Also for volatile periods, HS often overestimate risk and during periods of low volatility the method underestimate the risk.

### 3.4 ARMA-GARCH-EVT model

Empirical evidence confirm that the logarithms of returns series are not independently and identically distributed (McNeil et al.McNeil et al. (2015)). Before we can use EVT to model the tails of the distribution of an individual return series, we must ensure that the standardized residuals are approximately independent and identically distributed (iid). Therefore in order to address the deficiencies of the financial return series data we adopt a two step approach introduced by McNeil and Frey (2000). The conditional-EVT model suggests first to use a ARMA-GARCH model to filter the financial return series such that the residuals obtained are relatively close to satisfying the i.i.d. assumption of the original financial return series. In the second step, the POT based method is applied to model the tail behavior of standardized residuals obtained from the fitted ARMA-GARCH model. Consequently, the conditional EVT approach handles both dynamic volatility and heavy-tailed exhibited by the return distribution. The two-step approach can be summarized as follows:

**Step 1:** The ARMA-GARCH-type model assuming the innovations term follows a Student *t* distribution is fitted to the currency exchange return series.

**Step 2:** EVT is applied to the standardized residuals obtained in Step 1 to estimate the tail distribution. The POT method is used to select the exceedances of standardized residual beyond a high threshold.

The VaR forecast for the GARCH-type models rely on the one-day-ahead conditional variance forecast, σ 2 t+1 of the volatility model. To this extent, one-step ahead forecasts of the conditional variance of returns is recursively obtained as:

$$\hat{\sigma}^2_{t+1} = E\left(\sigma^2_{t+1}|F_t\right), \quad (28)$$

Where Ft is the information set at time t, and σ 2 t is defined as in the GARCH models. The rollingfixed-window estimation procedure is used to evaluate the out-of-sample performance of the GARCH-type models. In each window, the parameters of the GARCH-type models are estimated and then used to determine the one-step-ahead forecasts of the conditional mean, conditional variance and standardized residuals. For each GARCH-type model, under the assumption of different innovations term distribution the one-day-ahead VaR forecast at α% confidence level is obtained as:

$$\widehat{\text{VaR}}_{t+1}(\alpha) = \hat{\mu}_{t+1} + F^{-1}(\alpha)\hat{\sigma}_{t+1} \quad (29)$$

Where $F^{-1}$ (α) is the α-quantile of the cumulative distribution function of the innovations distribution. Before we can use EVT to model the tails of the distribution of an individual return series, we must ensure that the standardized residuals are approximately independent and identically distributed (iid). Therefore in order to address the deficiencies of the financial return series data we adopt a two step approach introduced by McNeil and Frey (2000). The conditional-EVT model suggests first to use a ARMA-GARCH model to filter the financial return series such that the residuals obtained are relatively close to satisfying the i.i.d. assumption of the original financial return series. In the second step, the POT based method is applied to model the tail behavior of standardized residuals obtained from the fitted ARMA-GARCH model. Consequently, the conditional EVT approach handles both dynamic volatility and heavy-tailed exhibited by the return distribution. The two-step approach can be summarized as follows:

**Step 1:** The ARMA-GARCH-type model assuming the innovations term follows a Student *t* distribution is fitted to the currency exchange return series. Step 2: EVT is applied to the standardized residuals obtained in

**Step 1:** to estimate the tail distribution. The POT method is used to select the exceedances of standardized residual beyond a high threshold.

For $q > F(\upsilon)$, $VaR_q$ can be obtained from equation (??) by solving for x which is given as;

$$VaR_q = \hat{\upsilon} + \frac{\hat{\beta}}{\hat{\xi}}\left[\left(\frac{n}{N_\upsilon}(1-q)\right)^{-\hat{\xi}} - 1\right] \quad (30)$$

Where υ is a threshold, β^ is the estimated scale parameter and ^ξ is the estimated shape parameter. The main advantage of unconditional GDP approach is that it focuses attention directly on the tail of the distribution. However, it does not recognize the fact that returns are non-independent and identically distributed.

### 3.5 Backtesting

In order to check whether the results obtained from the Value at Risk estimation are consistent and reliable, each model is verified by backtesting technique to determine whether the number of exceptions generated have come close enough to the realized outputs, with the help of statistical methods. Backtesting refers to testing the accuracy of VaR over a historical period when the true outcome is known. Various methods of backtesting are proposed. The first test is known as test of unconditional coverage of (Kupiec, 1995). The second test is conditional coverage of Christoffersen (1998) which examines the dependence in the data.

In order for backtesting to be implemented an indicator function of VaR violations also referred to as "exceedance or hit" is defined. Let $N = \sum_{t=1}^{T} I_{t+1}$ be the number of days over a T period that the portfolio loss was larger than the VaR estimate, where $I_{t+1}$ is an indicator of VaR violations that can be described as:

$$I_{t+1} = \begin{cases} 1, \text{if} & r_{t+1} < \text{VaR}_{t+1}|t \\ 0, \text{if} & r_{t+1} \geq VaR_{t+1}|t \end{cases} \tag{31}$$

#### 3.5.1 Unconditional Coverage Test

Kupiec (1995) suggested the unconditional coverage test (UC) for assessing whether the frequency of failures in the sample is in line with the expected number of violations (hits or exceedances) of the predicted VaR models.This test is also known as the Proportion of failures (POF) test. The likelihood ratio test developed by Kupiec (1995) was used. This test examines whether the failure rates is statistically equal to the expected one. Let p be the expected failure rate, (p = 1−q), where q is the confidence level for the VaR). If the total number of such trials is T, then the number of failures N can be modelled with a binomial distribution with probability of occurrence equals to α. The null and alternative hypothesis are, $H_1$: N/T = p and $H_1$: N/T ≠ p. The appropriate likelihood ratio statistic is:

$$LR_{uc} = 2\ln\left[\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}\right] - 2\ln\left[(1-p)^{T-N}p^{N}\right] \sim \chi^2_{k-1} \tag{32}$$

Where k is the possible number of outcomes, which is failure or success. Therefore, the LRuc ∼ $\chi^2$ (1) under H0 of good specification. This back-testing procedure is a two sided test. Hence, a model is rejected if it generates too many or too few violations, but based on it, the risk manager can accept a model that generates dependent VaR violations. The unconditional coverage test only classifies VaR model as adequate but does not account for the possibility of clustering of violations caused by volatile return series. It only tests whether the empirical frequency of violations (exceedances) is close to the prespecified VaR level. It does not test whether several quantile exceedances occur in rapid succession or whether they tend to be isolated.

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### 3.5.2 Conditional Coverage Test

This a more comprehensive and elaborate test proposed by Christoffersen (1998) which jointly investigates if, first, the total number of failures is equal to the expected one and second, the VaR failure process is independently distributed through time. That is, it tests for correct coverage and detecting whether a quantile violation today influences the probability of a violation tomorrow. This test provides an opportunity to detect VaR measures which are deficient in one way or the other. Under the null hypothesis that the failure process is independent and the expected proportion of violations is equal to p, the appropriate likelihood ratio is;

$$LR_{cc} = -2\ln\left[\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^{N}\right] + 2\ln\left[(1-\pi_{01})^{n_{00}}\,\pi_{01}^{n_{01}}\,(1-\pi_{11})^{n_{10}}\,\pi_{11}^{n_{11}}\right] \sim \chi^2(2) \quad (33)$$

where $n_{ij}$ represents the number of times an observation with value $i$ is followed by an observation with value $j$ for $i, j = 0$ and $\pi_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$ are the corresponding probabilities. The values $i, j = 1$ denote that a violation has been made, while $i, j = 0$ indicates the violation has not been made.

The main advantage of this test is that it can reject a VaR models that generates either too many or too few clustered violations, but it requires a number of observations to become more accurate. Non-rejection of the hypothesis leads to confidence in the reliability of the VaR model in predicting events of losses while rejection means that the model is not adequate in maintaining allowable VaR violations and is vulnerable to VaR exceedance clustering.

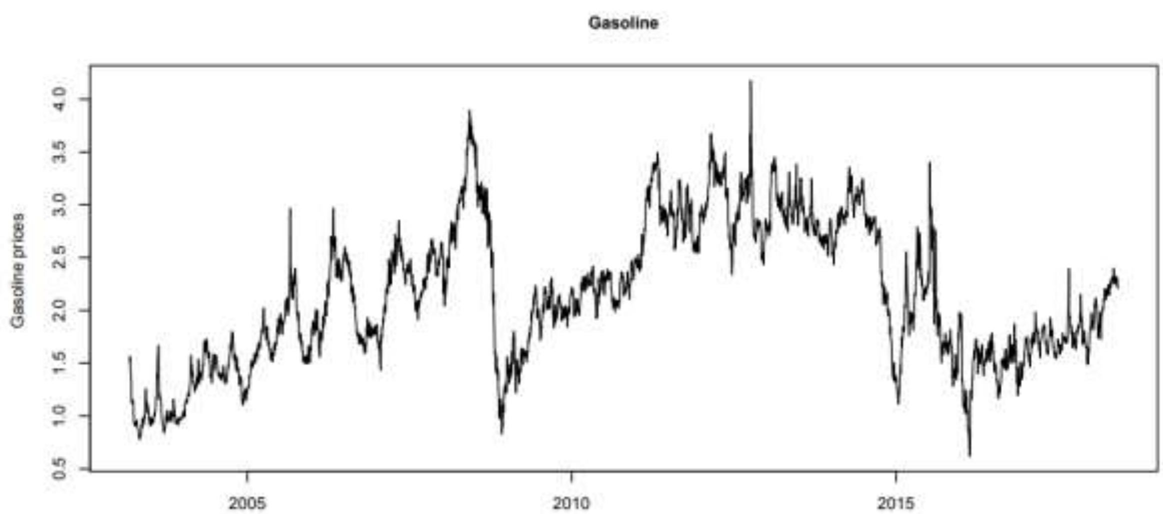
## 4.0 Empirical Results

### 4.1 Data and Data description

The data set consists of the daily spot prices of WTI Crude Oil (Dollars per Barrel) and Reformulated Regular Gasoline (Dollars per Gallon) for the period running from 11th March, 2003 to 14th June, 2018. The data consists of 3828 observations excluding weekends and public holidays. The data was obtained from the US Energy Information Administration (EIA) website provided by the THOMSON REUTERS source accessed on 14th June, 2018. First, the data is converted to daily log return series by logarithmic transformations given by $r_t = ln(p_t\, p_{t-1})$, where pt is the daily spot price at time $t$.

The time plot of energy data for daily spot prices of Crude Oil and Gasoline against time are given in Figure 1. The plot illustrates that prices changes exhibit volatility clustering with occasional jumps and spikes for both crude oil and gasoline where upward movements are followed by upward movements and downward movements followed by downward movements. The trend and patterns for both gasoline and crude oil prices gives almost similar volatility clustering except that gasoline prices indicated sharp spikes in its clusters compared to crude oil. Figure 2 presents the return plots for crude oil and gasoline spot prices. The returns are also characterized by patterns of time-varying volatility clustering where periods of high (low) volatility are followed by periods of high (low) volatility. The time-varying behaviour of crude oil and gasoline returns suggests the presence of stylized characteristics normally exhibited by financial time series data. The summary descriptive statistics and statistical tests results for the daily returns of both crude oil and gasoline are presented in Table 1. The statistics include the maximum, minimum, mean, standard deviation, skewness, kurtosis, Jarque-Bera statistics, Ljung-Box statistics for raw and squared returns. The descriptive summary statistics shows that gasoline has a minimum return lower than that of crude oil while crude oil has a maximum return value that is lower than that of gasoline. The standard deviation of crude oil is greater than standard deviation of gasoline from the measure of skewness, all the series for crude and gasoline spot prices returns are skewed to the right. These series exhibit positive excess kurtosis, and these are some of the stylized facts observed in financial time series data. Based on the p-values of the JB test, we reject the null hypothesis of normality for all the differenced series of crude oil and gasoline spot prices. Both series have positive means and are mean-stationary since the returns are concentrated around around zero as indicated in Figure 2. These series exhibits leptokurtosis as their kurtosis are greater than the normal kurtosis value of 3.

To check for any serial correlations in the log returns, Ljung-Box test is used. The p-value for each return series was less than 0.05. Therefore the null hypothesis is rejected (no serial correlation) and hence there is serial correlation in the log returns of both gasoline and crude oil. Similarly ARCH effect in the log-returns was tested where the Q-statistic values had p-values close to zero in both situations which shows that ARCH effect is present in the log returns of gasoline and crude oil.



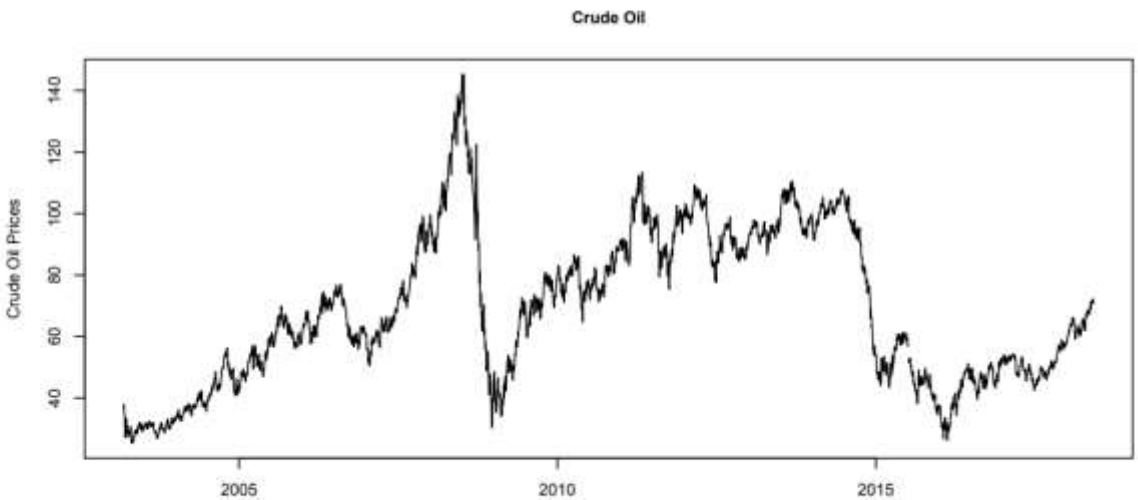
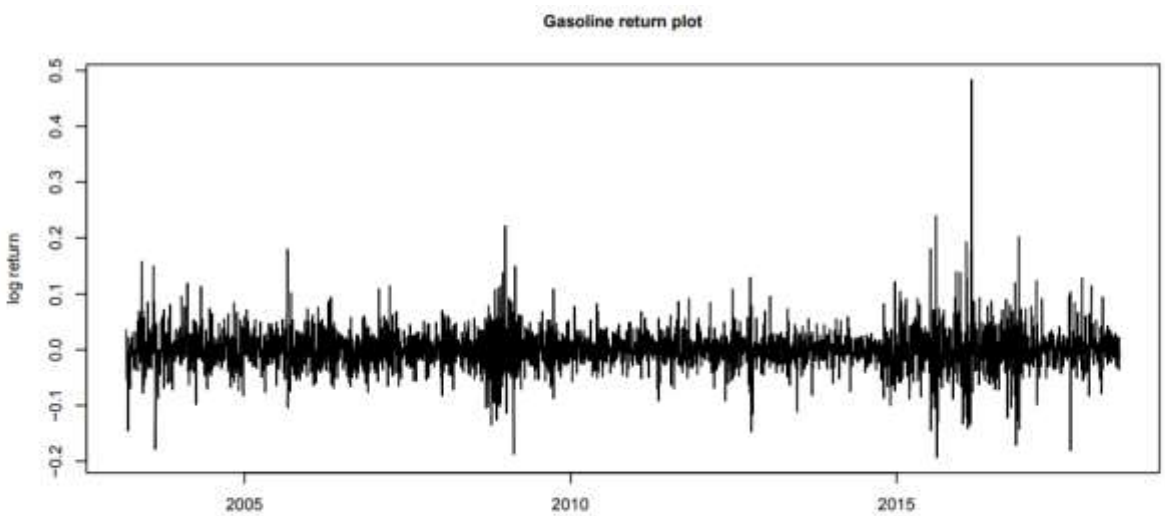



**Figure 1: Daily closing prices of Gasoline and Crude Oil (period from 11th March, 2003 to 14th June, 2018**



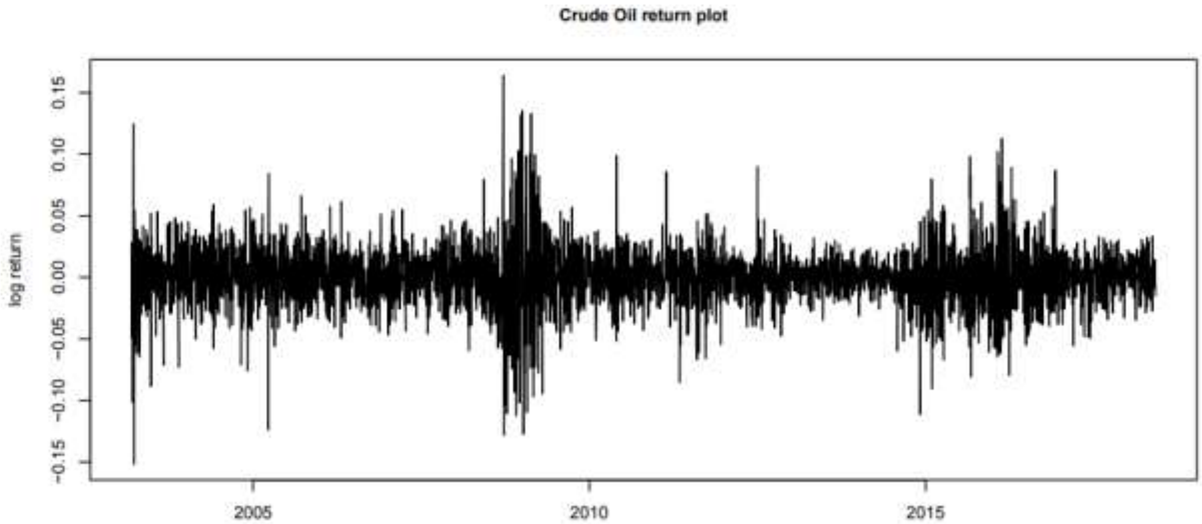



**Figure 2: Daily returns of Gasoline and Crude Oil (period from 11th March, 2003 to 14th June, 2018**

**Table 1: Descriptive Statistics and Tests of Daily Energy Log Returns**

Table 1: Descriptive Statistics and Tests of Daily Energy Log Returns

Descriptive Summary Statistics and Tests of Energy Returns

| Returns | No. of. Obs | Min | Max | Mean | Stdev | Skewness | Kurtosis | JB-Test (Prob) | ADF-Test | ARCH-Test | Ljungâ“Box Q^2(20) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Gasoline | 3823 | -0.193274 | 0.483797 | 0.000100 | 0.034491 | 0.912704 | 14.655487 | 34788 (0.0000) | -41.9535 (0.01) | 1538.7 (0.0000) | 387.85 (0.0000) |
| Crude Oil | 3823 | -0.151909 | 0.164137 | 0.000171 | 0.023942 | 0.000230 | 4.519787 | 3259.8,(0.0000) | -45.1012 (0.01) | 3577.7 (0.0000) | 3039.4 (0.0000) |

Values in brackets are respective p-values for the statistic

In performing the stationarity test, Augmented Dickey- Fuller (ADF) test was used. The test statistic is given by the p-value obtained from the results of ADF test. From the results obtained using ADF test on log returns gave in both cases of gasoline and crude oil a p-value of 0.01 which is less than 0.05 hence there was sufficient evidence at 5% significance level to reject null hypothesis of non-stationarity, which implied that the log returns are stationary under the ADF test as given in Table 1.

Table 2: The AIC values for the fitted ARMA (p, q) processes

| Gasoline Returns | | | | | |
|---|---|---|---|---|---|
| **Models** | ARMA (0, 0) | ARMA (0, 1) | ARMA (1, 1) | ARMA (1, 2) | ARMA (2, 2) |
| **AIC** | **-14894.25** | -14893.38 | -14892.99 | -14893.39 | -14891.82 |
| **Crude Oil Returns** | | | | | |
| **Models** | ARMA (0, 0) | ARMA (0, 1) | ARMA (1, 1) | ARMA (1, 2) | ARMA (2, 2) |
| **AIC** | -17685.7 | -17689.05 | -17687.17 | -17686.17 | **-17698.7** |

### 4.2 Parameter estimates of the GARCH Models

In this study, seven GARCH-type models: the SGARCH, IGARCH, GARCH-M, EGARCH, GJR-GARCH, APARCH, and FIGARCH models are utilized to model the conditional volatility and estimate one-step-ahead VaR forecast of the gasoline and crude oil spot prices. Further, two backtesting measures: the conditional and unconditional coverage tests are used to evaluate the out-of-sample VaR forecasts performance of the seven GARCH models. Prior to implementing the comparative performance of VaR forecast for the above GARCH models, the fitting of the implemented seven models is explored via the empirical results of the parameter estimates for the competing models. The fitting of the model involves determining the mean equation and volatility equations of the log returns.

#### 4.2.1 Parameter estimates of the Mean Component

The parameter estimation of the mean component is performed on the full-sample using a maximum likelihood estimator by considering different combinations of the parameters (p, q) ranging from zero to two. The best fitting ARMA models for the mean components are selected via the Akaike Information Criterion (AIC). The ARMA (p, q) specification with the smallest AIC value is selected as the best fitting model for the mean component of the energy commodities. The information criteria values for the fitted ARMA (p, q) models assuming the Student-t innovations distributions are reported in Table 2. The best mean equation for gasoline is ARMA (0, 0) with AIC of -14894.25 and that for crude oil is ARMA (2, 2) with AIC of -17698.7 among the fitted models.

#### 4.2.2 Parameter estimates of the Volatility Component

For the conditional volatility component we decided to analyze with the more parsimonious GARCH (2, 2) model. The ARMA part describes the dynamics of the trend (expected loss, conditional mean) and the GARCH part reflects the stochastic behaviour of the conditional volatility of losses. The best specification for the mean component is chosen based on the minimum value for the Akaike information criterion (AIC) and Bayesian Information Criterion (BIC). The most appropriate GARCH-type model is selected from the different specifications (GARCH, IGARCH, EGARCH, GJRGARCH, APARCH, TGARCH, NGARCH, NAGARCH, AVGARCH, FIGARCH and HGARCH) fitted to the crude oil and gasoline energy commodities assuming error terms follows the normal distribution, Student t and skew Student t distributions.

Table 3 presents the results for three information criteria: Akaike (AIC) and Schwartz (BIC) and Hannan-Quinn (HQC) for the fitted GARCH-type models. The GARCH (1, 1) with normal distribution had an AIC of -4.147930, GARCH (1, 1) assuming Student t had an AIC of -4.237750 and finally that under skew Student t with an AIC of -4.239647. Therefore, the fitted error term was skew Student t distribution which had the smallest AIC of -4.239647. The ARMA(0, 0)-GARCH(1, 1) model was selected as the best fitting mean model for gasoline returns. For Crude oil returns ARMA (2, 2)-GARCH (1, 1) model was selected as the best fitting model. The estimates of the combined models for both gasoline and crude oil are summarized in Tables 3 and 4 respectively with their extensions. The skew Student t distribution was considered as the innovations distribution for the error term.

The persistence of GARCH (1, 1) model of the coefficients is close to one indicating that volatility shocks are quite persistent, but not explosive. β, the parameter for the conditional variance and α1 are highly significant for the skewed Student t implying that the GARCH models make more accurate estimate of the variance. In GARCH extensions, IGARCH (1, 1) has persistence of 1 hence volatility shocks are highly persistent in the returns of gasoline. In all other extensions fitted, volatility persistence is close to unity, therefore an indicator that shocks in energy markets dies out very slowly which is a feature of long memory in the energy markets (this means that volatility is highly persistent and there is evidence of near unit root behavior of the conditional variance process).

Table 3: Parameter estimates of the mean and volatility component for the gasoline Log returns

**Gasoline spot price returns: Assuming skew Student t distribution: ARMA (0, 0)**

| MODELS | GARCH (1, 1) | IGARCH (1, 1) | GARCH-M (1, 1) | APARCH (1, 1) | EGARCH (1, 1) | GJR-GARCH (1, 1) | FIGARCH (1, 1) |
|---|---|---|---|---|---|---|---|
| $\mu$ | 0.000841 | 0.000947* | 0.004732* | 0.000700 | 0.000682 | 0.000793 | 0.000848 |
| $\alpha_0$ | 0.000037* | 0.000027* | 0.000038* | 0.000603 | -0.185940* | 0.000037* | 0.000058* |
| $\alpha_1$ | 0.098320* | 0.118276* | 0.096191* | 0.111360* | -0.009152 | 0.090365* | 0.279339* |
| $\beta_1$ | 0.873572* | 0.881724 | 0.873549* | 0.888185* | 0.973231* | 0.871831* | 0.579256* |
| $\lambda$ | | | -0.142364* | | | | |
| $\gamma$ | | | | 0.049651 | 0.199481* | 0.018787 | |
| $\delta$ | | | | 1.141220* | | | 0.436721* |
| skew | 1.067360* | 1.070230* | 1.057103* | 1.061393* | 1.059965* | 1.066380* | 1.064955* |
| shape | 4.528815* | 3.971576* | 4.502971* | 4.571688* | 4.545579* | 4.520869* | 4.377438* |
| Persistence | 0.9718925 | 1 | 0.9697399 | 0.9699712 | 0.9732314 | 0.9718672 | |
| AIC | -4.2396 | -4.2375 | -4.2408 | -4.2424 | -4.2427 | -4.2394 | **-4.2480** |
| BIC | -4.2298 | -4.2293 | -4.2293 | -4.2293 | -4.1951 | -4.2279 | **-4.2407** |

Parameter estimates marked with an asterisk (*) are significant at the 5% confidence level.

Table 4: Parameter estimates of the mean and volatility component for crude oil Log returns

| Crude Oil spot price returns: Using skew Student t: ARMA (2, 2) | | | | | | | |
|---|---|---|---|---|---|---|---|
| MODELS | GARCH (1, 1) | IGARCH (1, 1) | GARCH-M (1, 1) | APARCH (1, 1) | EGARCH (1, 1) | GJR-GARCH (1, 1) | FIGARCH (1, 1) |
| $\mu$ | 0.000481 | 0.000463 | -0.000054 | -0.000050 | 0.000197 | 0.000303 | 0.000482 |
| $\alpha_0$ | 0.000003 | 0.000002 | 0.000003 | 0.000093 | -0.062969* | 0.000003 | 0.000002* |
| $\alpha_1$ | 0.057274* | 0.060890* | 0.056975* | 0.052476* | -0.056677* | 0.023594* | 0.049027 |
| $\beta_1$ | 0.937414* | 0.939110 | 0.937635* | 0.950897* | 0.991995* | 0.943312* | 0.947408* |
| $\lambda$ | | | 0.030241 | | | | |
| $\gamma$ | | | | 0.629449* | 0.102664* | 0.055197* | |
| $\delta$ | | | | 1.134554* | | | 0.999999* |
| skew | 0.940920* | 0.939148* | 0.941500* | 0.924747* | 0.928916* | 0.935281* | 0.939799* |
| shape | 8.004111* | 7.551278* | 8.043256* | 8.355618* | 8.596203* | 8.347869* | 7.713796* |
| Persistence | 0.9946884 | 1 | 0.99461 | 0.9931198 | 0.993842 | 0.993842 | |
| AIC | -4.9452 | -4.9450 | -4.9447 | -4.9547 | **-4.9567** | -4.9512 | -4.9451 |
| BIC | -4.9288 | -4.9225 | -4.9268 | -4.9351 | **-4.9303** | -4.9332 | -4.9271 |

Parameter estimates marked with an asterisk (*) are significant at the 5% confidence level.

The symmetrical GARCH-M (1, 1) model was estimated by allowing the mean equation of the return series to depend on a function of the conditional variance. The estimated coefficient of risk premium in Table 3 of the σ 2 t in the mean equation is negative for the gasoline returns which indicates that the mean of the return sequence does not depend on past innovations and conditional variance. These results show that as volatility increases, the returns corresponds by decrease with a factor of -0.142364 in gasoline returns. Hence these results are not consistent with the theory of a positive risk premium on asset returns which states that higher returns are expected for assets with higher level of risk.

For the APARCH (1, 1) model in Table 3, the estimated parameters are not all statistically significant at 5% level of significance. The asymmetric (leverage) effect captured by parameter estimate γ has a positive sign indicating that its significant at 5% level and therefore leverage effect exists in the returns. This shows that positive shocks imply a higher next period of conditional variance than negative shocks of the same sign which indicates the existence of leverage effects in the returns of gasoline in energy markets. This model fits in this returns of gasoline.

The asymmetrical EGARCH (1, 1) estimated parameters of gasoline returns in Table 3 are all significant at 5% confidence level except the ARCH parameter and mean which is insignificant at 5% level. The leverage effect parameter is statistically significant and have a positive sign indicating that positive and negative shocks have different effects on volatility in the gasoline returns from energy market. Hence this model also fits well in this spot data of gasoline. Other extensions which were estimated is the GJR-GARCH (1, 1) model. From table 3, all estimated parameters are significant at 5% level except the mean and leverage effect parameter which is not significant at 5% confidence level. This indicates that GJR-GARCH (1, 1) model is not adequate in capturing leverage effect for long memeory data of energy markets. Lastly, FIGARCH (1, 1) model was estimated and gave all parameter estimates significant at 5% level except that the leverage parameter and persistence were unavailable therefore, its conclusion was not reached.

For crude oil spot price series, GARCH and its extensions were also estimated as shown in Table 4. The GARCH (1, 1) had coefficient persistence close to a unity indicating high volatility shocks in the data. Other extensions had too persistence close to one which is an indication of long memory in energy market data. For IGARCH (1, 1) in Table 4, coefficient persistence was a unity indicating high volatility shocks in crude spot data returns. GARCH-M (1, 1) model captured risk premium parameter being positive indicating returns have a positive correlation on its volatility. Increase in volatility corresponds to increase in the returns by a factor which is 0.03024. Therefore the model is adequate at 5% level of significance. The APARCH (1, 1), EGARCH (1, 1) and GJR-GARCH (1, 1) models estimated gave leverage effect parameter with positive sign and significant too at 5% level. This indicates that the positive and negative shocks in the returns have different effects on volatility of the returns of the crude oil spot price returns from energy market. The skewness parameter for the fitted models of the GARCH and the specified GARCH extensions is highly significant in both cases of the gasoline and crude oil returns. From the estimated parameters, the best fitted models based on AIC were ARMA (0, 0)-EGARCH (1, 1) for gasoline returns and ARMA (2, 2)-EGARCH (1, 1) for crude oil returns since they gave significant results overall.

Value-at-Risk Forecasts The VaR is calculated at 95% and 99% confidence levels. Backtesting period equals 1000 days. Value at Risk estimates are given as aggregate VaR for the portfolio of gasoline and crude oil, stand-alone VaR and contribution VaR for the normal, HS, FHS and IGARCH (using EWMA with decay factor, lambda of 0.94) models. The models are used compared with the help of Backtesting which is performed for the whole holding period. Best models which estimate VaR more accurately than other models is recommended. VaR using fitted GARCH type models are used as a benchmark in Backtesting procedure of these methods. EVT Value at Risk is also estimated and its parameters extracted. The results of the VaR estimations at 95% for the case of all models is shown in Table 5.

Table 5: VaR for the Normal, HS, FHS and IGARCH at 95% confidence level

| | Normal | | Historical Simulation | | Filtered Simulation | | IGARCH/EWMA | |
|---|---|---|---|---|---|---|---|---|
| **Returns** | Stand-alone VaR | VaR contribution | Stand-alone VaR | VaR contribution | Stand-alone VaR | VaR contribution | Stand-alone VaR | VaR contribution |
| **Gasoline** | 0.6409375 | 0.02464831 | 0.3761537 | -0.006571154 | 0.4995999 | 0.01712644 | 0.7719471 | 0.013424064 |
| **Crude Oil** | 0.3590625 | 0.01380834 | 0.6238463 | -0.010898178 | 0.5004001 | 0.01715388 | 0.2280529 | 0.003965811 |
| Aggregate VaR | | **0.03845665** | | **0.03493866** | | **0.03428032** | | **0.01738988** |

Aggregate VaR estimate of IGARCH gives the smallest value compared to other models.

The stand-alone VaR for each position is the VaR that the portfolio would have if all other positions are ignored. The VaR contribution provides a measure of risk for each individual portfolio position that takes into account correlations between risk factors. They are calculated so

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that the sum of VaR contributions for all positions equals the VaR of the whole portfolio. Lastly, the aggregate VaR of the portfolio is the total VaR for individual result. VaR estimates are also calculated at 99% confidence level and results are given in Table 6. The IGARCH model performs well in both 95% and 99% confidence levels in estimating VaR.

Table 6: VaR for the Normal, HS, FHS and IGARCH at 99% confidence level

| | Normal | | Historical Simulation | | Filtered Simulation | | IGARCH | |
|---|---|---|---|---|---|---|---|---|
| **Returns** | Stand-alone VaR | VaR contribution | Stand-alone VaR | VaR contribution | Stand-alone VaR | VaR contribution | Stand-alone VaR | VaR contribution |
| **Gasoline** | 0.6406565 | 0.03488119 | 0.3761537 | -0.01185047 | 0.8196418 | 0.06503723 | 0.93397118 | 0.022800782 |
| **Crude Oil** | 0.3593435 | 0.01956482 | 0.6238463 | -0.01965386 | 0.1803582 | 0.01431112 | 0.06602882 | 0.001611943 |
| Aggregate VaR | | **0.05444601** | | **0.06300867** | | **0.07934835** | | **0.02441273** |

IGARCH model still gave the smallest value of aggregate VaR at 99% confidence level.

The extreme value theory method was used to estimate VaR of the two portfolios as a single value at 95% and 99% confidence level and its parameters were extracted too which is shown in Table 7. The shape parameter is greater than zero hence the distribution of gasoline and crude oil belongs to the Frechet family. The VaR estimates for EVT at 95% and 99% confidence levels for aggregate VaR are greater than that of IGARCH in Tables 5 and 6. This still leaves IGARCH model as the best in estimating VaR for gasoline and crude oil.

Table 7: GPD parameters and VaR using EVT at 95% and 99% confidence levels

| | **Extreme Value Theory (EVT) VaR** | | | | |
|---|---|---|---|---|---|
| | 95% | 99% | Threshold (u) | Scale parameter ($\xi$) | Shape parameter ($\beta$) |
| **VaR estimate** | 4.420265 | 8.298004 | 0.04431987 | 0.2595738 | 0.01944079 |

**4.3 Backtesting VaR Forecasts**

Backtesting is used to verify consistency of a given VaR model with actual losses. The main idea of Backtesting VaR model is that outcomes predicted by the model are compared with the actual trading results. The day when realized losses fall below the VaR estimation is known as exceedances or hits. According to Jorion *et al.* (2007), Kupiec developed confidence intervals of the non-rejection region for number of exceedances at 95% confidence level which acts as a benchmark at different periods.

The outcome of backtesting for portfolio returns of gasoline and crude oil using GARCH type models shows the number of exceedances for each VaR model, with an alpha level set at 5% and 1%. The conditional coverage test and unconditional coverage test critical values are 5.991465 and 3.841459 respectively. The results containing statistics and exceedances at each significance level are given in Table 8. The backtest of the GARCH type are used as a benchmark in comparing the accuracy of the VaR models. Table 8, the critical values for unconditional coverage test is same for all three models as well as the critical values of the conditional coverage test. The statistics for all the three models at 5% level for unconditional coverage and conditional coverage are less than the critical values. This is because the exceedances are less than 65 for 1000 days as holding period for backtesting and according to Jorion *et al*. (2007) is failed to be rejected at 5% level. All the three GARCH type models in estimating VaR gives significant results. The backtest results of the whole period length of 1000 observations with a one day moving window of gasoline and crude are important in this technique. All the returns are plotted and, as observed, some observations have returns lower than the Value at Risk level which is 5%. These observations are called exceedances and are marked red in the Figures 4 and 3. The black line represents the VaR level forecasted for a period length of 3823 observations. The Crude oil experiences many exceedances which are spread away from the VaR level in Figure 3 as compared to that of Gasoline in Figure 4. The variation of the VaR lines follow different trends since VaR for crude oil has many upwards and downwards movements with sharp spikes which accumulates less exceedances as to that of gasoline VaR which accumulates more exceedances on its limits. The unconditional coverage test and conditional coverage test are used to analyse the VaR Models accuracy at two confidence levels of 95% and 99%. The sample size of 3823 observations have been used for parameter estimation for Gasoline and Crude Oil log returns. For backtesting purposes of the VaR models, the last four years sample data was used for backtesting hence T=1000 days for both sets of log returns. The results of the expected number of exceedances versus actual exceedances after backtesting for each of the VaR models are illustrated in Tables 9 and 10. The VaR Normal VaR model and EVT at 1% level for both log return series accumulate many exceedances over the back-test period, while at 5% level, EVT models more exceedances for gasoline return series. Normal VaR results are not surprising since our data exhibited some excess kurtosis that cannot be captured by the normal VaR model.

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Table 8: Kupiec and Christoffersen coverage tests of the sGARCH (1, 1), EGARCH (1, 1) and GARCH-M (1, 1) at 95% confidence interval

| Backtest Evaluation | sGARCH (1, 1) | EGARCH (1, 1) | GARCH-M (1, 1) |
|---|---|---|---|
| Backtest Length | 1000 | 1000 | 1000 |
| Alpha | 5% | 5% | 5% |
| Expected Exceed | 50 | 50 | 50 |
| Actual VaR Exceed | 62 | 64 | 60 |
| **Unconditional Coverage (Kupiec) 95% confidence level** | | | |
| Null-Hypothesis | Correct Exceedances | | |
| LR.uc Statistic | 2.800674 | 3.775802 | 1.963121 |
| LR.uc Critical | 3.841459 | 3.841459 | 3.841459 |
| LR.uc p-value | 0.09422467 | 0.05199896 | 0.1611799 |
| Reject Null | NO | NO | NO |
| **Conditional Coverage (Christoffersen)** | | | |
| Null-Hypothesis | Correct Exceedances & Independence of Failures | | |
| LR.cc Statistic | 2.807787 | 3.990427 | 2.011873 |
| LR.cc Critical | 5.991465 | 5.991465 | 5.991465 |
| LR.cc p-value | 0.2456388 | 0.1359846 | 0.365702 |
| Reject Null | NO | NO | NO |

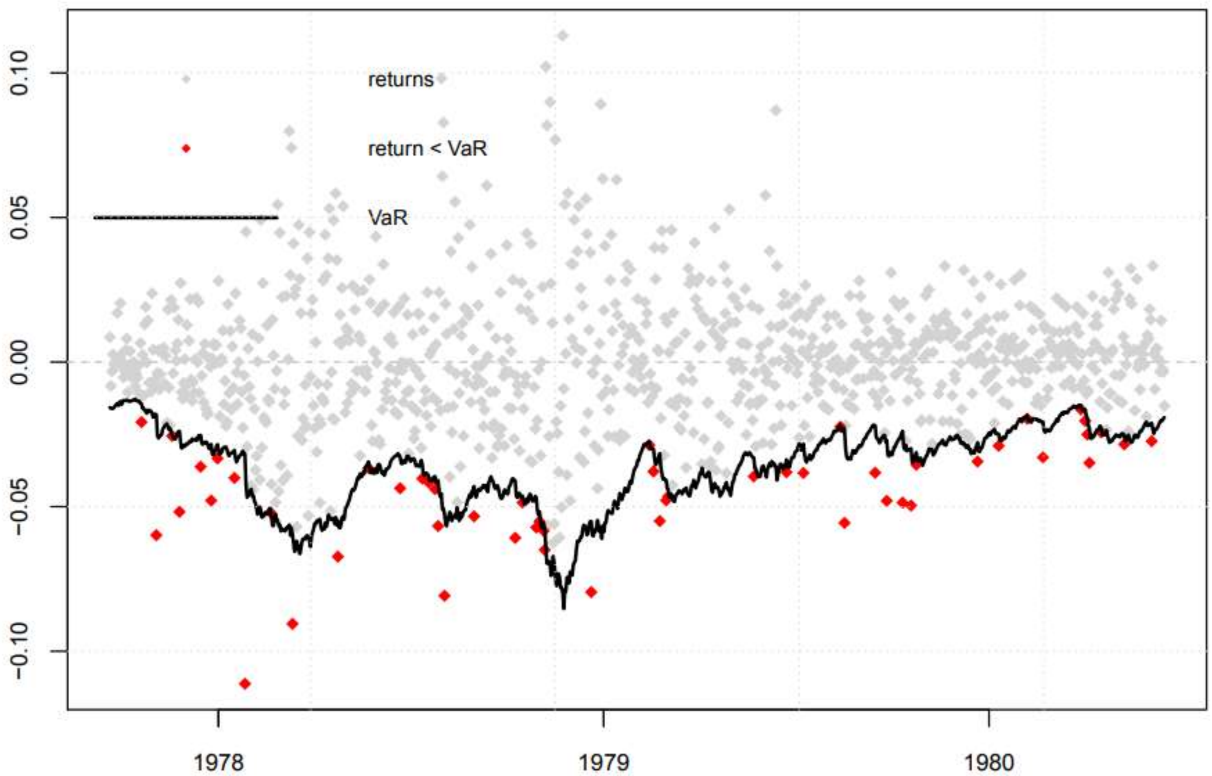


**Figure 3: Backtesting Results at 95% Value at Risk for Crude Oil returns**

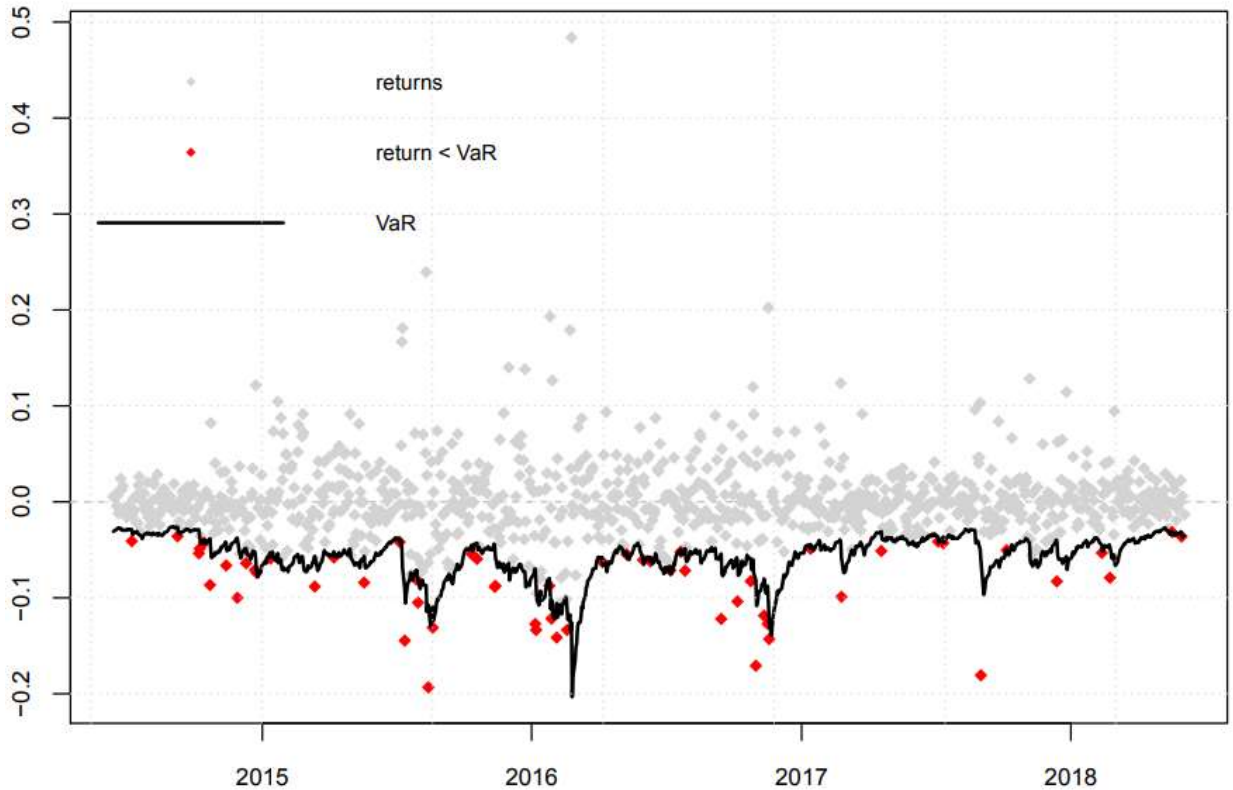


**Figure 4: Value-at-Risk forecasts at 95% level of significance for Gasoline returns**

The VaR models at 1% level for gasoline return series that accumulated lesser exceedances are IGARCH, GARCH-t and GARCH-skew t models and for crude oil series, only HS had smaller number of exceedances at 1% level. At 5% level, IGARCH appeared to have lesser exceedances compared to other VaR models for gasoline return series while EGARCH-skew t under crude oil return series accumulated lesser exceedances. Thus, there is an indication that Normal VaR model and EVT at 1% for two sets of series do not perform well as other models since they underestimate Value at Risk due to accumulation of many exceedances hence their violation ratios is greater than expected ratio. HS and EVT VaR models at 5% level for gasoline and crude oil returns underestimates VaR too because of accumulating a lot of exceedances. However, HS VaR model at 1% level for crude oil accumulates exceedances less than the expected hence indicates that VaR violation ratio is less than expected ratio and therefore, showed that VaR is underestimated for this series when HS is used. The IGARCH/EWMA performed well for both return series at 1% and 5% levels.

Table 9: Backtesting of VaR models for Gasoline Log Returns

| Gasoline Log Returns | | | | | | |
|---|---|---|---|---|---|---|
| $\alpha$ | 1% | | | 5% | | |
| **Model** | Expected Exceed | VaR Exceed | Actual % | Expected Exceed | VaR Exceed | Actual % |
| **Normal** | 10 | 35 | 3.5% | 50 | 71 | 7.1% |
| **HS** | 10 | 25 | 2.5% | 50 | 77 | 7.7% |
| **IGARCH** | 10 | 11 | 1.1% | 50 | 40 | 4% |
| **GARCH-t** | 10 | 11 | 1.1% | 50 | 51 | 5.1% |
| **GARCH-skew t** | 10 | 11 | 1.1% | 50 | 56 | 5.6% |
| **EGARCH-t** | 10 | 12 | 1.2% | 50 | 57 | 5.7% |
| **EGARCH-skew t** | 10 | 13 | 1.3% | 50 | 60 | 6% |
| **EVT** | 10 | 32 | 3.2% | 50 | 79 | 7.9% |

Table 10: Backtesting of VaR models for Crude Oil Log Returns

| **Crude Oil Log Returns** | | | | | | |
|---|---|---|---|---|---|---|
| $\alpha$ | 1% | | | 5% | | |
| **Model** | Expected Exceed | VaR Exceed | Actual % | Expected Exceed | VaR Exceed | Actual % |
| **Normal** | 10 | 16 | 1.6% | 50 | 52 | 5.2% |
| **HS** | 10 | 6 | 0.6% | 50 | 56 | 5.6% |
| **IGARCH** | 10 | 12 | 1.2% | 50 | 50 | 5% |
| **GARCH-t** | 10 | 15 | 1.5% | 50 | 55 | 5.5% |
| **GARCH-skew t** | 10 | 14 | 1.4% | 50 | 54 | 5.4% |
| **EGARCH-t** | 10 | 14 | 1.4% | 50 | 52 | 5.2% |
| **EGARCH-skew t** | 10 | 11 | 1.1% | 50 | 49 | 4.9% |
| **EVT** | 10 | 4 | 0.4% | 50 | 30 | 3.0% |

However, under both coverage tests for crude oil indicates that Normal VaR model cannot be rejected even though its exceedances are higher than expected number of exceedances. The results for other models shows that the null hypothesis holds at 1% level for all models except HS, and EVT since neither of them have a likelihood ratio statistic surpassing the critical value from the chi-square statistic. This is also indicated by the p-values which are above the 5% level associated with 95% confidence level of the test. At 5% level, EVT, GARCH-t and GARCH-skew-t does not satisfy the conditional coverage test for crude oil return series which shows that these models are not independent in VaR estimation. Under the gasoline return series, the Normal, HS and EVT VaR models does not satisfy both coverage tests and therefore, cannot perform well in estimating VaR for crude oil because of underestimating of VaR. IGARCH model performs well under unconditional coverage test but in using conditional coverage test shows that its not independent since at 5% level, the null hypothesis does not hold. Therefore, for both return sets under unconditional and conditional tests, EGARCH-t and EGARCH-skew t performs well in VaR estimation since they all satisfy the two tests at every level of test. The results are given in Tables 11 and 12.

Furthermore, for gasoline return series, EGARCH-t and GARCH-skew t VaR models have similar test statistics which implies that either of them can be used in forecasting VaR. The econometric VaR models for these two series under unconditional coverage test holds for their null hypotheses at 1% and 5% levels only that they does not all satisfy the property of independence under conditional coverage test as shown in Tables 11 and 12. Therefore, we conclude that EGARCH-t and EGARCH-skew t VaR models under econometric category perform better in forecasting VaR. IGARCH/EWMA VaR model performs better too under parametric models since the null hypothesis holds for both coverage tests at 1% and 5% levels for both return sets and therefore its good in VaR estimation.

Therefore, in analysis of Value at Risk models, IGARCH, EGARCH-t and EGARCH-skew-t models performed best among the eight VaR models used. They were followed by GARCH-t and GARCH-skew t models and those VaR models that performed worse was Normal, EVT and HS models.

Table 11: Kupiec and Christoffersen Tests for Crude Oil Log Returns

| Test | Uncondtional Coverage test | | | | Conditional Coverage test | | | |
|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 1% | | 5% | | 1% | | 5% | |
| **Models** | LR. statistic | P-value | LR. statistic | P-value | LR. statistic | P-value | LR. statistic | P-value |
| **Normal** | 3.08 | 0.08 | 0.08 | 0.77 | 3.60 | 0.17 | 0.31 | 0.86 |
| **HS** | 103.12 | 0.00 | 0.73 | 0.39 | 103.65 | 0.00 | 1.26 | 0.53 |
| **IGARCH\EWMA** | 0.38 | 0.54 | 0.00 | 1 | 0.67 | 0.71 | 5.27 | 0.07 |
| **GARCH-t** | 2.19 | 0.14 | 0.51 | 0.48 | 2.65 | 0.27 | 6.92 | 0.03 |
| **GARCH-skew t** | 1.44 | 0.23 | 0.33 | 0.57 | 1.84 | 0.40 | 6.50 | 0.04 |
| **EGARCH-t** | 1.44 | 0.23 | 0.08 | 0.77 | 1.84 | 0.40 | 5.80 | 0.06 |
| **EGARCH-skew t** | 0.10 | 0.75 | 0.02 | 0.88 | 0.34 | 0.84 | 5.08 | 0.08 |
| **EVT** | 4.71 | 0.03 | 9.77 | 0.02 | 4.74 | 0.09 | 9.78 | 0.01 |

Table 12: Kupiec and Christoffersen Tests for Gasoline Log Returns

| Test | Uncondtional Coverage test | | | | Conditional Coverage test | | | |
|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 1% | | 5% | | 1% | | 5% | |
| **Models** | LR. statistic | P-value | LR. statistic | P-value | LR. statistic | P-value | LR. statistic | P-value |
| **Normal** | 38.33 | 5.97 | 8.26 | 0.00 | 45.73 | 1.17 | 27.28 | 0.00 |
| **HS** | 16.04 | 6.19 | 13.27 | 0.00 | 25.13 | 3.49 | 27.99 | 0.00 |
| **IGARCH\EWMA** | 0.01 | 0.75 | 2.25 | 0.31 | 0.34 | 0.84 | 5.09 | 0.08 |
| **GARCH-t** | 0.01 | 0.75 | 0.02 | 0.89 | 0.34 | 0.84 | 0.09 | 0.96 |
| **GARCH-skew t** | 0.10 | 0.75 | 0.73 | 0.39 | 0.34 | 0.84 | 0.74 | 0.69 |
| **EGARCH-t** | 0.38 | 0.54 | 0.99 | 0.32 | 0.67 | 0.72 | 1.01 | 0.60 |
| **EGARCH-skew t** | 0.83 | 0.36 | 1.98 | 0.16 | 1.17 | 0.56 | 2.03 | 0.36 |
| **EVT** | 30.93 | 0.00 | 15.17 | 0.00 | 36.48 | 0.00 | 28.61 | 0.00 |

## 5.0 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

### 5.1 Summary

Modeling and forecasting volatility of the energy markets is of importance in the world economy. It helps the industry players to manage risks that might occur market and risk managers will also be able to plan well when information on volatility is available and investors on the other hand will be able to invest wisely in energy commodities. In this article we modeled energy market volatility using GARCH models and estimated Value-at-Risk using GARCH-EVT model and other conventional models. The data set consisted of daily observations over the period of 11th March, 2003 to 14th June, 2018 for Gasoline and Crude Oil commodities. The empirical results suggests that among the APARCH (1, 1), EGARCH (1, 1) and GJR-GARCH (1, 1) models, the leverage parameter was only significant for EGARCH (1, 1) model at 5% level hence being able to capture leverage effect in the log returns of both sets. All the other GARCH models used in modeling volatility had persistence close to one implying that volatility shocks in energy market commodities dies out slowly. The best fitted model among those fitted was EGARCH (1, 1) model under the error term of skew Student t distribution.

Among the volatility models fitted, the EGARCH (1, 1) and ARMA (2, 2)-EGARCH (1, 1) were the best models in estimating volatility of gasoline and crude oil respectively. This is because EGARCH (1, 1) model had the smallest AIC and leverage parameter was significant at 5% level among the fitted models.

### 5.2 Conclusion

Therefore, the study concluded that EGARCH (1, 1) model is the best in fitting and volatility modeling of the energy commodities of gasoline and crude oil. Thee performance of VaR models is evaluated based on unconditional coverage and conditional coverage tests. In VaR performance and backtesting, the results showed that all the fitted models had VaR violation ratios greater than expected ratio indicating that VaR is underestimated which implies that the models recommends for less capital allocation. The VaR models that performed best were IGARCH/EWMA, EGARCH-t and EGARCH-skew t models. They gave optimal results compared to other used VaR models because of passing both coverage tests.

The Normal VaR model, HS, IGARCH model, EVT and econometric models of GARCH and EGARCH were used in VaR estimation and backtesting. The Normal VaR model, HS and EVT did not perform well in VaR estimation since they did not satisfy both coverage tests and there VaR violation ratios were greater than the expected ratios, thereby indicating underestimation of VaR. The IGARCH, EGARCH-t and EGARCH-skew t VaR models performed well since they all satisfied both unconditional and conditional coverage tests, therefore, they are also independent in estimating Value at Risk.

### 5.3 Recommendations

In light of the research findings, the study recommends that organizations should leverage modern technology as a basis of realizing efficiency, effectiveness, and sustainability of projects. The study likewise recommends that organizations should build capacities to enhance labour productivity. In addition, the study recommends that organizations should adopt transformational leadership approaches as a basis of enhancing performance. The study recommends the need to revise the legal framework with a view to ensure that it reflects the changing needs of the project requirements.

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