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Research Article

A Jump Diffusion Model with Fast Mean-Reverting Stochastic Volatility for Pricing Vulnerable Options

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The Black–Scholes–Merton option pricing model is a classical approach that assumes that the underlying asset prices follow a normal distribution with constant volatility. However, this assumption is often violated in real-world financial markets, resulting in mispricing and inaccurate hedging strategies for options. Such discrepancies may result into financial losses for investors and other related market inefficiencies. To address this issue, this study proposes a jump diffusion model with fast mean-reverting stochastic volatility to capture the impact of market price jumps on vulnerable options. The performance of the proposed model was compared under three different error distributions: normal, Student-*t*, and skewed Student-*t*, and under different market scenarios that consist of bullish, bearish, and neutral markets. In a simulation study, the results show that our model under skewed Student-*t* distribution performs better in pricing vulnerable options than the rest under different market scenarios. Our proposed model was fitted to S&P 500 Index by maximum likelihood estimation for the mean and volatility processes and Gillespie algorithm for the jump process. The best model was selected based on AIC and BIC. Samples of the simulated values were compared with the S&P 500 values and MSE computed at various sample sizes. Values of MSE at different sample sizes indicate significant decrease to actual MSE values demonstrating that it provides the best fit for modeling vulnerable options.

1. Introduction

Over-the-counter (OTC) markets have grown significantly in recent years, raising concerns about default risk, particularly following the 2008 global financial crisis. Since OTC options are not subject to regular market reconciliation and margin replenishment, they expose option holders to increased credit risks. These options, which are vulnerable to counterparty credit risk, are known as vulnerable options. Furthermore, OTC markets are dominated by trend traders, whose tendency to follow market trends often leads to market congestion, resulting in the whipsaw effect influencing the price discovery of options that are traded in this market. During such periods, increased price volatility disrupts pricing inputs like implied volatility, making it challenging to accurately determine option prices. In response to these challenges and with potential applications in controlling counterparty credit risk and facilitating smoother negotiation processes for these options during market congestion, research for improved pricing models has emerged as an increasingly vital topic in finance. These efforts aim to mitigate pricing inefficiencies and enhance risk management for OTC options in today's dynamic financial landscape.

Black and Scholes [1] pioneered the concept of option pricing models in their seminal paper by assuming that the underlying asset price follows a geometric Brownian motion with a constant mean and volatility and that the asset return series follows a normal distribution. However, this model has been exposed to criticism and limitations due to these assumptions. The assumption that the underlying asset follows a geometric Brownian motion with a constant mean and volatility means that the price of the underlying asset moves continuously and follows a smooth and predictable path over time. However, empirical studies have shown that the underlying asset's price curve is not smooth but has jumps. The other assumption that the underlying asset return series follows a normal distribution contradicts the empirical studies because asset returns' distribution has leptokurtic features implying that the distribution of asset returns has a higher peak and asymmetric heavier tails than those of the normal distribution. While these assumptions make the model more tractable, they create limitations when applied to real-world financial markets. Such limitations include option mispricing during extreme events such as market crashes, inadequacy in capturing market realities such as volatility skew, inaccurate prediction of discontinuous payoffs, and significant losses to financial institutions that rely heavily on this model for risk management.

Ki et al. [2] proposed a closed pricing formula for European options the case where the return of the underlying asset follows extended normal distribution for different degrees of skewness and kurtosis relative to the normal distribution. Numerical experiments and a comparison of the empirical performance of the proposed model with the Black–Scholes model were done through the estimation of implied parameters such as standard deviation, skewness, and kurtosis of the return on the underlying asset from the market prices of the KOSPI 200 Index options. The results demonstrated that the actual density of the underlying asset depicts skewness to the left with high peaks. Similarly, Burger and Kliaras [3] empirically investigated the Black–Scholes model and the Merton model both constructed regarding normal distribution and the double exponential jump diffusion model which does not assume a normal distribution of the stock returns but a distribution that has got a higher peak and two heavier tails and also considers the empirical abnormity called volatility smile. The empirical results showed that the double exponential jump diffusion model fitted the stock data better as compared to the other two models. Gatheral et al. [4] noted that the Black–Scholes approach underestimates the probability of extreme events in asset returns' distribution due to its constant volatility assumption. Moreover, Liu et al. [5] utilized a model of stochastic volatility featuring jumps within the price of the underlying asset and the counterparty asset value to derive solutions for the option price. Their findings indicated that the stochastic volatility model with jumps can provide more accurate pricing compared to the Black–Scholes model. Similarly, Zhou et al. [6] considered an improved model for pricing vulnerable options by incorporating the dynamics of the underlying asset and counterparty asset as a class of jump diffusion processes. These findings challenge the model's validity in real-world scenarios and emphasize the need for more sophisticated approaches to option pricing and risk management. Besides the jump diffusion model employed in this study, other diffusion processes have been explored in existing literature, such as self-diffusion and cross-diffusion, as evident in recent studies (see Chen et al. [7], Chen and Wu [8], Chen and Srivastava [9], and Zhu et al. [10]).

This paper proposes to study a jump diffusion model under the Student-*t* and skewed Student-*t* distributions instead of the Gaussian distribution. The dynamics of the underlying asset price, option writer's asset value, and stochastic volatility are derived, and the pricing formula of the vulnerable options is obtained. The application of the proposed model is demonstrated by considering three different error distributions (normal, Student-*t*, and skewed Student-*t*) and three market trends (bullish, bearish, and neutral) and performing simulations of the model under the different market trends and error distributions. Empirical results are obtained by fitting the proposed model using the S&P 500 Index prices under the three different error distributions. Performance of the proposed model was tested by computing the AIC and BIC and the mean square error under different sample sizes. By addressing these aspects, this study aims to present a more accurate and robust approach to pricing vulnerable options, offering valuable insights into risk management and option pricing strategies for financial practitioners.

The rest of this paper is organized as follows. In Section 2, we describe the methodology of pricing vulnerable options using the jump diffusion model with fast mean-reverting stochastic volatility. Section 3 discusses the simulation results obtained from the application of this methodology. Section 4 presents the empirical results. Finally, in Section 5, we provide the conclusions and suggestions for further research.

2. Methodology

In financial markets, the jumps in financial asset prices are normally triggered by policy changes, catastrophic events, and big news events. In this section, the jump diffusion model with mean-reverting stochastic volatility is derived.

Assume that $\mathcal{T} = [0, T]$ is the time level and $(\Omega, F, \{F_t\}_{t\in\mathcal{T},P})$ is the complete probability space where P is the physical probability measure. The dynamics of the underlying asset price S_t and the option writer's asset value V_t are assumed to be described by the following stochastic differential equations (SDEs), respectively:

$$\begin{aligned} \frac{\mathrm{d}S_t}{S_t} &= \mu_1 + \sigma_1 dB_t^1, \\ \frac{\mathrm{d}V_t}{V_t} &= \mu_2 + \sigma_2 dB_t^2, \end{aligned} \tag{1}$$

where μ_1 and μ_2 are drift components of underlying asset price and option writer's value, respectively, σ_1 is the volatility of the underlying asset price, σ_2 is the volatility of the option writer's value, and B_t^1 and B_t^2 are the standard Brownian motions of the underlying asset price and option writer's value, respectively.

To account for jumps in the underlying asset price and option writer's asset value, a jump process is introduced to the right hand side of equation (1). Therefore, the dynamics of S_t and V_t follow a jump diffusion process given by

$$dS_t = \mu S_{t\_}dt + Y_t S_{t\_} dB_t^S + S_{t\_} d\left(\sum_{i=1}^{N_{1t}} \left(e^{\xi_i} - 1\right)\right) - \lambda_1 \beta_1 S_{t\_} dt,$$
$$dV_t = \mu V_{t\_}dt + Y_t V_{t\_} dB_t^V + V_{t\_} d\left(\sum_{j=1}^{N_{2t}} \left(e^{\nu_j} - 1\right)\right) - \lambda_2 \beta_2 V_{t\_} dt, \tag{2}$$

where $S_{t\_}$ and $V_{t\_}$ represent the value of S_t and V_t just before a possible jump of sizes e^{ξ} and e^{ν} at time t, respectively. N_{1t} and N_{2t} denote the counts of the observed number of jump times prior to time t with intensities λ_1 and λ_2, respectively. $\beta = \beta_1 = \beta_2$ represent the expected jump size conditional on information available at time t. Within this model represented by equation (2), μ, Y_t, and $(\sum_{i=1}^{N_{1t}} (e^{\xi_i} - 1)) - \lambda_1\beta_1$ represent the conditional mean, the stochastic volatility process, and the jump process of the underlying asset, respectively. For model estimation, the conditional mean is fitted by estimating the parameters by the MLE method and then residuals are obtained from the fitted model, and volatility process is also fitted by estimating parameters from the residuals by the MLE method. Simultaneously, we use the Gillespie Algorithm to estimate the jump process..

The percentage increase in the price of the underlying asset and option writer's asset value given the price changes from S_{t-} to $e^{\xi}S_t$ and V_{t-} to $e^{\nu}V_t$, respectively, is given by

$$\frac{e^{\xi}S_t - S_{t-}}{S_{t-}} = \frac{\Delta S_t}{S_{t-}} = e^{\xi} - 1,$$
$$\frac{e^{\nu}V_t - V_{t-}}{V_{t-}} = \frac{\Delta V_t}{V_{t-}} = e^{\nu} - 1, \tag{3}$$

where $\Delta S_t = \Delta V_t \longrightarrow dt$ as $\Delta t \longrightarrow 0$ is the infinitesimal limit dt. Therefore, the total number of jumps is given by

$$J_i = \sum_{i=1}^{N_{1t}} \left(e^{\xi_i} - 1\right),$$
$$J_j = \sum_{i=1}^{N_{2t}} \left(e^{\nu_i} - 1\right). \tag{4}$$

To ensure that S_t and V_t are martingales, the jump components in equation (2) are compensated by $\lambda_1\beta_1 dt$ and $\lambda_2\beta_2 dt$, respectively.

Let Y_t denote the stochastic volatility process assumed to follow the Ornstein–Uhlenbeck process given by

$$dY_t = \frac{1}{\varepsilon}(m - Y_t)dt + \frac{u\sqrt{2}}{\sqrt{\varepsilon}} dB_t^Y, \tag{5}$$

where ε denotes the inverse of the mean reversion rate and $Y_t \sim N(m, u^2)$.

To obtain an arbitrage-free price of the vulnerable option, a risk-neutral measure P^* from Johannes and Polson [11] is introduced in equations (2) and (5) to get

$$dS_t = rS_{t\_}dt + Y_t dB_t^{S^*} S_{t\_} + S_{t\_} d\left(\sum_{i=1}^{N_{1t}} \left(e^{\xi_i} - 1\right)\right) - \widetilde{\lambda}_1 \widetilde{\beta}_1 S_{t\_} dt,$$
$$dV_t = rV_{t\_}dt + Y_t dB_t^{V^*} V_{t\_} + V_{t\_} d\left(\sum_{j=1}^{N_{2t}} \left(e^{\nu_j} - 1\right)\right) - \widetilde{\lambda}_2 \widetilde{\beta}_2 V_{t\_} dt, \tag{6}$$
$$dY_t = \left(\frac{1}{\varepsilon}(m - Y_t)\frac{u\sqrt{2}}{\sqrt{\varepsilon}}\Lambda(Y_t)\right)dt + \frac{u\sqrt{2}}{\sqrt{\varepsilon}} dB_t^{Y^*},$$

where $B_t^{S^*}, B_t^{V^*}, B_t^{Y^*}, \widetilde{\lambda}_1, \widetilde{\beta}_1$, and $\Lambda(Y_t)$ are defined under P^*.

According to Klein [12], the payoff of a vulnerable European call option at time T is given by

$$C(S_T, V_T) = (S_T - K)^+ \left[1_{V_T \geq \widetilde{D}} + \frac{V_T(1-\alpha)}{D} 1_{V_T < \widetilde{D}}\right], \tag{7}$$

where $\widetilde{D}$ is the default boundary, D is the total claim value, and α is the dead-weight cost of financial distress expressed as a percentage of option writer's asset value.

Therefore, the price of a vulnerable call option at time $t \leq T$ is given by

$$\mathbb{P}(t, s, \nu, y) = E^{P^*}\left[e^{-r(T-t)}C(S_T, -S_T) \,\middle|\, S_t = s, -S_t = \nu, Y_t = y\right], \tag{8}$$

where $E^{P^*}[\cdot]$ is the conditional expectation under the risk-neutral measure P^*.

As a consequence, the solution of $\mathbb{P}(t, s, v, y)$ is given by a PDE in Oksendal [13] with the terminal condition

$$\mathbb{P}(T, s, v, y) = (s - K)^+ \left[1 \middle| (-S_T \geq \widetilde{D}) + \frac{-S_T(1-\alpha)}{D} \middle| (-S_T < \widetilde{D}) \right]. \quad (9)$$

Given the terminal condition in equation (9), obtaining an analytical solution for this partial differential equation (PDE) is not feasible due to its complexity. As a result, numerical simulations of the proposed model were performed. The Gillespie algorithm was used to simulate the price jumps. We defined three distinct states representing the bullish (BU), bearish (BE), and neutral (NE) market trends, each associated with specific rates governing the occurrence of market price jumps. These rates were calibrated to reflect the intensity of jumps in each market state. We denote the state vector $x = (x_1, x_2, x_3)$ where x_1, x_2, x_3 represents the counts of BE, BU, NE, respectively. Table 1 lists the interactions between different states, propensity functions of reactions, and the net vectors. From Table 1, if the market experiences an event that leads to (1) a downward jump in stock prices, the economy moves from BU to BE and is modeled in reaction R_1, (2) an upward jump in stock prices, the economy moves from BE to BU and is modeled in reaction R_2, (3) and no jump in stock prices, the economy in either BU or BE moves to NE and is modeled in reactions R_3, R_4, R_5, and R_6.

To implement the Gillespie algorithm, we adapted the steps outlined in Altıntan et al. [14].

3. Simulation Results

In this section, we illustrate the application of the jump diffusion model with the Gillespie algorithm, as presented in equation (2). The Gillespie algorithm was selected for its unique features, including its ability to simulate individual reactions rather than the system as a whole, which results in a more precise and accurate simulation of the underlying stochastic processes. Moreover, its flexibility in handling complex models with multiple stochastic variables and events, combined with its computational efficiency and suitability for large datasets, makes it ideal for real-time simulations. These features make the Gillespie algorithm a powerful tool for simulating complex systems, such as the jump process in the proposed model.

Figure 1 illustrates the comparison of the adapted model and the fitted model under three different error distributions.

Table 2 presents a set of parameters adapted from Turchyn [15] and Altıntan et al. [14] based on real market data. The adapted set of parameter estimates in Table 2 was used to fit the proposed jump diffusion model with fast mean-reverting stochastic volatility.

The parameters of the proposed model were estimated using the maximum likelihood estimation method. Table 3 presents the parameter estimates of the fitted model using three distinct residual distributions, namely, normal, Student-t, and skewed Student-t, under three market trends (bearish, bullish, and neutral). The parameter estimates were found to be independent of the market trends (bearish, bullish, and neutral). This implies that the proposed model is appropriate to price vulnerable options as it will provide the jump process of the underlying asset regardless of whether the market is bullish, bearish, or neutral.

Table 1: Reaction list and reaction propensities of the jump process.

Reaction list	Reaction propensity
R_1: BE $\longrightarrow^{c}$ 1 BU	$\alpha_1 = c_1 x_1$
R_2: BU $\longrightarrow^{c}$ 2 BE	$\alpha_2 = c_2 x_2$
R_3: BU $\longrightarrow^{c}$ 3 NE	$\alpha_3 = c_3 x_2$
R_4: NE $\longrightarrow^{c}$ 4 BU	$\alpha_4 = c_4 x_3$
R_5: BE $\longrightarrow^{c}$ 5 NE	$\alpha_5 = c_5 x_1$
R_6: NE $\longrightarrow^{c}$ 6 BE	$\alpha_6 = c_6 x_3$

c_i describes the stochastic reaction rate of moving from one state to the other constant of the reaction R_i for $i = 1, 2, \ldots, 6$.

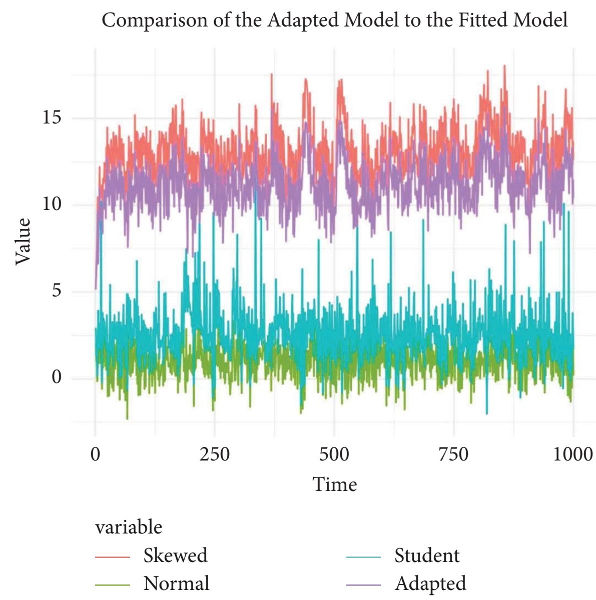

Figure 1: Comparison of the adapted model to the fitted model under skewed Student-t, Student-t, and normal distributions.

4. Empirical Results

This section presents the empirical results and discussions of the results obtained from fitting the proposed model to the dataset consisting of 4410 daily average closing prices of the S&P 500 Index, covering the period from 1st January 2005 to 31st July 2022. The data exclude weekends and holidays downloaded from https://www.investing.com. In Figure 2, the time series plot of the daily prices depicts the trend of the prices over time, while the log returns plot highlights the stochastic volatility dynamics in the daily prices. Significant price jumps are observed during the 2007-2008 global

Table 2: Parameter estimates adapted from Turchyn [15] and Altıntan et al. [14] based on real market data.

Parameter	μ	ϕ_1	ω	α_1	β_1	c_1	c_2	c_3	c_4	c_5	c_6
Value	0.033015	−0.09861	0.01142	0.065437	0.894563	0.2	0.5	0.6	0.2	0.5	0.6

Table 3: Parameter estimates of the fitted model using three different error distributions (normal, Student-t, and skewed Student-t) under three different market trends (bearish, bullish, and neutral).

Residual distribution	μ	ϕ_1	ω	α_1	β_1	Shape	Skew	c_1	c_2	c_3	c_4	c_5	c_6
Normal	0.033561	−0.111893	0.012502	0.087566	0.905233			0.2	0.5	0.6	0.2	0.5	0.6
Student-t	0.05335	−0.086657	0.00833	0.091376	0.907624	6.3745		0.2	0.5	0.6	0.2	0.5	0.6
Skewed Student-t	0.03343	−0.09033	0.00782	0.091132	0.907867	0.887993	6.75413	0.2	0.5	0.6	0.2	0.5	0.6

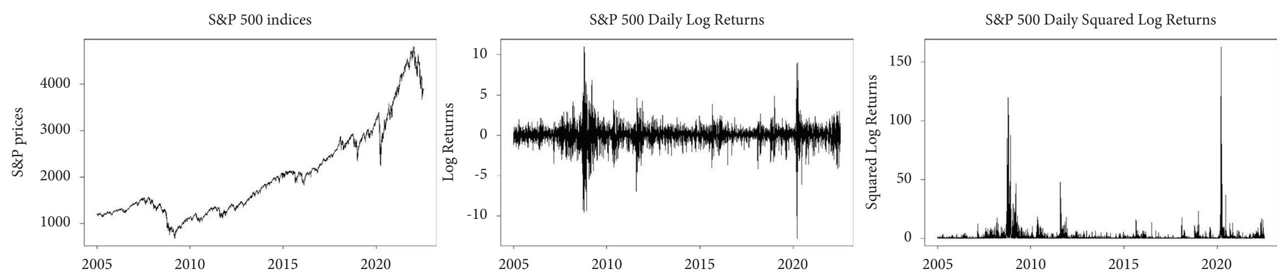

Figure 2: Times series, log return, and squared log return series of the S&P 500 Index for the period starting from 1st January 2005 to 31st July 2022.

financial crisis period and COVID-19 pandemic period. The market crashes caused the returns to exhibit extreme high asymmetric volatility and scattered jumps. The log returns also appear to fluctuate around the mean level, exhibiting volatility clustering, where large changes are followed by large changes, and small changes are followed by small changes. This implies that the log return exhibits conditional heteroskedasticity that can be modeled using conditional heteroskedastic models.

Table 4 presents descriptive summary statistics and statistical tests of the index prices, log returns, and squared log returns of the S&P 500 Index prices. The minimum and maximum values provide a range of the observed values in the data. The standard deviations are all positive giving an indication of the volatility of the underlying asset prices. The negative skewness of the log returns indicates that the distribution is negatively skewed, with the tail on the left side longer than the right side. Furthermore, the kurtosis of the log returns is greater than 3, indicating that the distribution is heavy tailed, and extreme values occur more frequently than in a normal distribution. These characteristics are consistent with the presence of volatility clustering and fat tails in the distribution of log returns. The Jarque–Bera (JB) test for normality confirms that the log returns are not normally distributed. The ARCH-LM test for the residuals of the log returns confirms the presence of heteroskedasticity. Therefore, the jump diffusion model with fast mean-reverting stochastic volatility could be useful in describing the dynamics of vulnerable options using the S&P 500 Index as the underlying asset.

Figure 3 illustrates the comparison between the fitted model under various error distributions and the S&P 500 Index. The fitted model's plot under the skewed Student-t distribution closely resembles the plot of the S&P 500 Index prices. These findings show that the proposed model under the skewed Student-t distribution could be a useful model for pricing vulnerable options.

The proposed jump diffusion model was fitted to the S&P 500 Index prices under the three error distributions. Table 5 presents the parameter estimates of the fitted model under different error distributions. The parameter estimates for both the conditional mean and volatility equations are confirmed to be statistically significant. In addition, the shape parameter and skewness parameter for both the Student-t and skewed Student-t distributions are statistically significant. Thus, the use of heavy-tailed innovation distribution seems justified to account for skewness and excess kurtosis in the asset returns.

To evaluate the relative goodness of fit of the proposed model under the different distributions to S&P 500 Index, the AIC and BIC of the proposed model were computed under various sample sizes. Table 6 presents the AIC and BIC values under different sample size values. The results demonstrate that as the sample size increases, skewed Student-t has the smallest AIC and BIC. This demonstrates that the proposed model with skewed Student-t distribution fits the return series more appropriately compared to the other two error distributions.

In order to test the performance of the proposed model under the three error distributions (normal, Student-t, and

Table 4: Summary statistics of prices, log returns, and squared log returns of S&P 500 Index.

Statistic	Min	Max	Std. dev	Skewness	Kurtosis	JB test	ARCH-LM test
Prices	676.50	4796.6	960.4607	1.080189	3.363511	882.47 (p-value = 0)	43,716 (p-value = 0)
Log returns	−12.76521	10.95720	1.239736	−0.560901	13.308883	7839.7 (p-value = 0)	1057.7 (p-value = 0)
Squared log return	0	162.95069	6.005256	13.1644	243.3772	10742319 (p-value = 0)	1106.4 (p-value = 0)

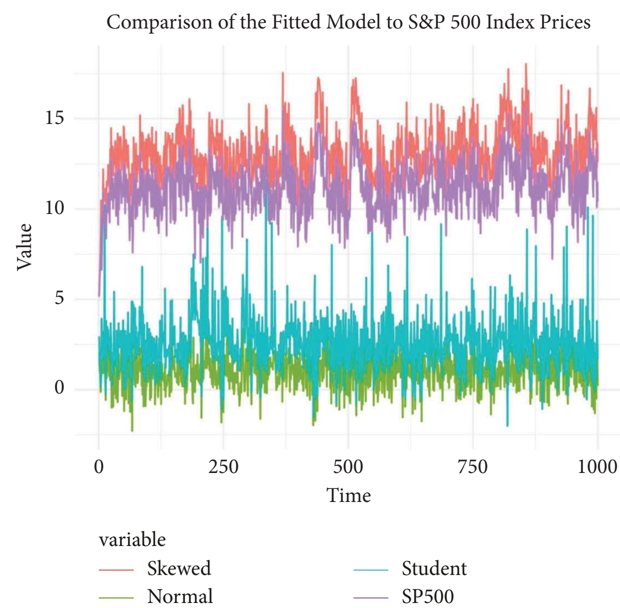

Figure 3: Comparison of the fitted model under skewed Student-t, Student-t, and normal distributions to the S&P 500 Index prices from April 2011 to October 2011.

Table 5: Parameter estimates of the proposed model fitted to the S&P 500 Index under the three error distributions.

Residual distribution	μ	ϕ_1	ω	α_1	β_1	Shape	Skew
Normal	0.068529	−0.073545	0.026622	0.142245	0.837764		
Student-t	0.084252	−0.0069566	0.016373	0.140577	0.857729	5.115944	
Skewed Student-t	0.060347	−0.0082497	0.0015629	0.133876	0.860204	0.874354	5.703997

Table 6: AIC and BIC of the proposed model under normal, Student-t, and skewed Student-t distributions for various sample sizes.

Sample size	Normal		Student-t		Skewed Student-t	
	AIC	BIC	AIC	BIC	AIC	BIC
1000	2.9466	2.9653	2.9017	2.9296	2.8968	2.9230
2000	2.9051	2.9191	2.8624	2.8792	2.8554	2.8750
3000	2.7080	2.7180	2.6615	2.6735	2.6531	2.6671
4000	2.6422	2.6501	2.5769	2.5863	2.5672	2.5782

skewed Student-t), the mean square error (MSE) was computed for both the fitted model and the S&P 500 Index prices. Table 7 presents the MSE values under different sample size values. The results demonstrate that as the sample size increases, the MSE of the proposed model decreases for all three distributions. This means that as the sample size increases, the suggested model's forecast of S&P 500 Index prices becomes increasingly accurate, and thus the model's prediction accuracy grows. These findings imply that when pricing vulnerable options using the proposed method, the distribution for the error component is an important factor in attaining accurate results. Overall, skewed Student-t distribution showed the lowest MSE value among the three distributions for all sample sizes. This demonstrates that the proposed model with skewed Student-t distribution fits the return series more appropriately compared to the other two error distributions. These findings support prior findings that, under the assumption

Table 7: Mean square error of the proposed model under normal, Student-t, and skewed Student-t distributions for various sample sizes.

Sample size	Normal MSE	Student-t MSE	Skewed Student-t MSE
1000	2.316982	2.213004	2.207537
2000	1.963498	1.941933	1.937141
3000	1.56498	1.548567	1.513999
4000	1.545911	1.517461	1.443388

of a skewed Student-t distribution, the suggested model gives a better fit for pricing vulnerable options.

5. Conclusions and Recommendation

This paper presents a jump diffusion model with fast mean-reverting stochastic volatility for pricing vulnerable options. The proposed model's application is illustrated by fitting the model using three distinct residual distributions, namely, normal, Student-t, and skewed Student-t, under three market trends (bearish, bullish, and neutral). From empirical results, the distribution of the log return series of S&P 500 Index was found to be negatively skewed and heavy tailed and this confirms the existing research that log returns of underlying asset of vulnerable options are not normally distributed. In addition, the proposed model with skewed Student-t distribution fits the return series of the S&P 500 Index more appropriately compared to normal distribution. To validate these results, future research can expand the analysis to different historical options datasets, explore other types of options, and compare this with other option pricing models.

Data Availability

The S&P 500 Index data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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