

Techniques for Evaluation and Selection of Partners for Construction Projects

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Abstract

Inter-enterprise collaborations require careful evaluations of partner enterprises and their attributes. Evaluation of partners for a project is a multi-criteria decision making process. The project initiator defines multiple criteria to be used in the selection of suitable partners. This study compares three different multi-criteria decision making techniques. Analytical Hierarchy Process (AHP) uses pairwise comparisons of crisp numerical values to derive weights of importance of partners. Fuzzy AHP (FAHP) uses pairwise comparisons of fuzzy values to derive weights of importance. Reduced Group Fuzzy AHP (RGFAHP) computes geometric mean of lower and upper bound fuzzy values to derive weights of importance. Eighty persons evaluated five companies to do structural engineering works for a large building. Their evaluation values were subjected to these algorithms. Total mean relative weights of partners were 0.9936, 0.9968 and 0.9866 with errors of 0.0064, 0.0032 and 0.0134 with time complexities of $n(n+6)$, $n(n-1)/2$ and $n(n-1)$ for AHP, FAHP and RGFAHP respectively.

AHP is effective when dealing with crisp evaluation values while FAHP is effective for fuzzy evaluation values. RGFAHP combines fuzzy approximate reasoning with conventional AHP, reduces the number of comparisons when a large number of attributes are used and deals with imprecise evaluators' judgement.

Keywords- Multi Criteria Decision Making (MCDM), Analytical Hierarchy Process (AHP), Fuzzy AHP (FAHP), Reduced Group Fuzzy AHP (RGFAHP), Partners Selection and Evaluation Problem (PSEP)

1. INTRODUCTION

Construction projects are implemented through a collaboration of different contractors supervised by a consultant. Delayed completion of projects [1], frequent collapse of buildings [2], use of inappropriate specifications and manuals, incompetent design, lack of ethics, poor supervision and use of inappropriate materials, poor coordination and management of contractors [3] are among the

challenges. This can be attributed to poor choice of partners for the tasks. Project initiators (partner evaluators) use different selection criteria and sub criteria to make choices among partners that are suitable for a particular task. There is need for a multi criteria decision making technique that can be used effectively by evaluators to determine the right partners. This study analyses different multi-criteria decision making algorithms for a partner selection problem.

2. PREVIOUS WORKS

The partner selection and evaluation can be considered a Multi-Criteria Decision-Making (MCDM) process, characterized by a substantial degree of uncertainty and subjectivity due to limited information about partners. Several multi-criteria decision making techniques have been proposed. Zhang, Liu and Van [4] considered a Weighted Sum Algorithm (WSA) [5] for the selection of partners. However, WSA is applicable only when all the data are expressed in exactly the same unit. Also its weighting coefficients do not necessarily correspond directly to the relative importance of the objectives or allow tradeoffs between the objectives to be expressed. Data Envelopment Analysis (DEA) [6,7,8,9] is a Linear Programming based technique for the analysis of efficiency of organizations with multiple inputs and outputs. In DEA, absolute efficiency cannot be measured, statistical tests are not applicable and large problems can be demanding. Elimination Et Choix Traduisant la Réalité (ELECTRE) [10] allows decision makers to select the best choice with utmost advantage and least conflict in the function of various criteria. The ELECTRE method is used for choosing the best action from a given set of actions. The decision maker uses concordance and discordance indices to analyze outranking relations among different alternatives and to choose the best alternative using crisp data. ELECTRE method is time consuming. The Technique for Order Preference by Similarity to Ideal

Solution (TOPSIS) method [11,12] assumes that each criterion has a tendency of monotonically increasing or decreasing utility which leads to easily defining the positive and the negative ideal solutions. The chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. TOPSIS is also time consuming.

Many research works have analyzed and solved multi-criteria decision making problems using multi-level analysis of alternatives. Analytical Hierarchy Process (AHP) [13] is a multi-criteria decision-making (MCDM) algorithm that uses pairwise comparisons of alternatives to derive weights of importance from a multi-level hierarchical structure of alternatives depending on the problem [13]. One of the shortcomings of AHP [14] is its inability to take into account any uncertainty associated with mapping human judgement to a number scale. Wang and Chin [15] found out that increase in the number of alternatives in each level of the hierarchy geometrically increases the number of pairwise comparisons by $O(n^2/2)$ which can lead to inconsistency or failure of the algorithm. Zadeh [5], Mikhailov [16] and Covella and Olsina [17] suggested the use of fuzzy logic to deal with subjectivity of the evaluators. Incorporation of fuzzy logic in multi-criteria decision making techniques can deal with shortcomings of AHP and improve the outcome of the partner selection and evaluation problem (PSEP).

A questionnaire (in the appendix) was given to 80 evaluators to indicate their preference of one company over another by examining their profiles. Section A of the questionnaire was used to indicate level of importance of each criteria (business, technical and management) against each other in the selection and evaluation process. The following sub criteria were rated against each other according to how they satisfied business criterion, financial security (FS), strategic position (SP) and business strength (BS). Likewise sub criteria technical capability (TC), development speed (DS), cost of development (CD) and information technology (IT) were rated according to how they satisfied the technical criterion. Finally, level of importance of sub criteria, collaboration record (CR), cultural compatibility (CC) and management ability (MA) in satisfying management criterion was given. Section B of the questionnaire was used to rate partners against each other according to how they satisfied each sub criterion. This information is represented in figure 1. To rate criteria and sub criteria, each evaluator chose alphabetical symbols (A, B, C, D, E) with matching linguistic attributes (extremely important, very important, important, weakly important and not at all important) respectively. The linguistic attributes for partners evaluation were (extremely preferable, very preferable, preferable, moderately preferable and not at all preferable).

3. METHODOLOGY

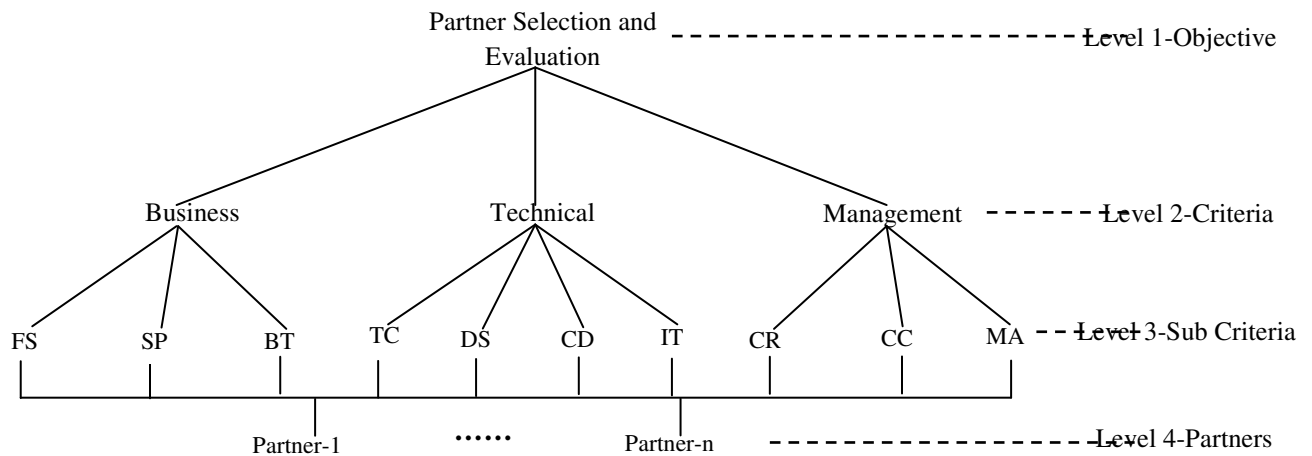


Figure1 Representation of the Partner Selection and Evaluation

Figure 1 illustrates that the problem was decomposed into a four level hierarchy of objective, selection criteria, sub criteria and partners. The process was simplified into finding the best partner for a structural engineering works of a building. This could be replicated to find best organizations for other tasks like electrical, mechanical & plumbing, interior design and landscaping works.

4. MULTI-LEVEL MULTI CRITERIA DECISION MAKING ALGORITHMS

This section describes the Multi-Criteria Decision Making Algorithms (MCDMA) used to evaluate and select partners. It begins with AHP followed by Fuzzy AHP and then Reduced Group Fuzzy AHP

(that has the characteristics of both AHP and Fuzzy AHP).

4.1 The Analytical Hierarchy Process

AHP method uses pairwise comparisons of values assigned by evaluators to alternatives (criteria, sub criteria and partners) in a multi-level hierarchical structure to derive their relative weights [13]. The hierarchical structure fits well with the hierarchical structure of partner evaluation and selection problem. According to Saaty [13] and Finnie et al. [18], AHP algorithm has the following steps: 1) Define the unstructured problem and state clearly the goal/objectives and outcomes; 2) Decompose the complex problem into a hierarchical structure of alternatives; 3) Employ pairwise comparisons and form pair-wise comparison matrices; 4) Use the

Eigenvalue method to estimate the relative weights; 5) Check the consistency of decision judgements; 6) Aggregate the relative weights to obtain the overall rating for alternatives. Figure 2 summarizes steps for AHP.

According to Vila and Beccue [19] and in the context of this study, the first step for AHP is to decompose a problem into a number of hierarchical levels. At the highest level of the hierarchy, the objectives are placed, then decision criteria are at the next level, and sub-criteria and partners are at the lowest level of the hierarchy. Each of the alternative is normally associated with a weight that indicates its significance in relation to other alternatives. Evaluators give their opinions on the importance of alternatives. From these opinions local and global priority weights are derived.

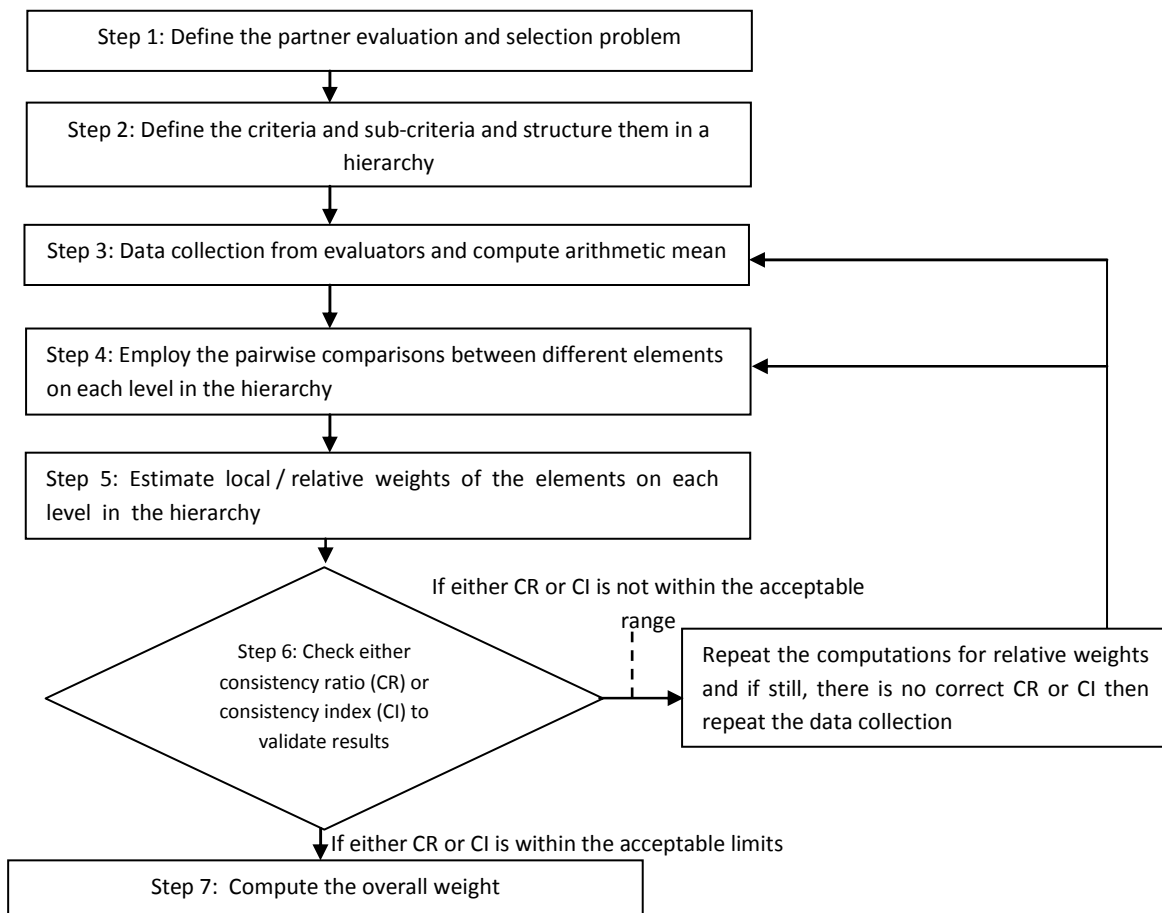


Figure 2: Steps of AHP

Local weights are relative weights of each alternative. Computation of local weights is performed through pairwise comparison of the alternatives, using the Saaty nine-point scale (Table 1). This results in so called, pairwise comparison matrices (PCM) of alternatives at the same level in the hierarchy.

Saaty [13] proposes that alternatives can be assigned a crisp (exact) value to show how important the alternative is viz a viz others. For example, if two alternatives have equal importance, each is assigned the numerical value 1 and if one alternative have moderate importance over the other, then it is assigned a numerical value 3. If one alternative is strongly or essentially important than another, it is assigned value 5, while value 7 is assigned to an

alternative that has very strong or demonstrated importance over another. If an alternative is

absolutely important than another, it is assigned

Table 1 Saaty Scale [13]

Definition	Intensity of importance
Equal importance	1
Moderate importance over one another	3
Essential or strong importance	5
Very strong or demonstrated importance	7
Absolute importance	9
Intermediate values between adjacent scales	2, 4, 6, 8

numerical value 9. Saaty [13] proposes the Eigenvalue method to compute pairwise comparison matrix and relative local weights. To explain Eigenvalue method, the following sections use data from evaluators for criteria. The averages of evaluators opinions after conversion from linguistic

to Saaty scale for the criteria, business, technical and management at level 2 of figure 1, were 9, 7, 7 respectively. Using these values, pairwise comparison matrix (PCM) for the level is computed as follows. Table 1 is the PCM for level 2.

Table 2 Pairwise Comparison Matrix for Level 2 in Figure 1

Criteria	Business	Technical	Management
Business	1.00	1.286	1.286
Technical	0.778	1.000	1.000
Management	0.778	1.000	1.000
Sum	2.556	3.286	3.286

To get the values in PCM, divide numerical value of one criterion by value of another criterion. Business skills criterion against itself is $9/9=1$, business skills against technical skills is $9/7=1.286$, while business skills against management skills is $9/7=1.286$. Technical skills against itself and management skills against itself is equally important with $7/7=1$. Technical skills against management is $7/7=1$. These are the values in the upper diagonal of the Table 2. Values in the lower diagonal of the table are the

reciprocals of the respective values in the upper diagonal. In table 2, technical skills against business is the reciprocal of business skills against technical which is $1/1.286=0.778$. The same case is applicable with management skills against technical which is $1/1=1$. Values in table 2 are then normalized by dividing the values of each field in a column by the sum of the values in the specific column. This results in values in Table 3.

Table 3 Normalized PCM, Priority Vectors and Local Weights for Criteria

Criteria	Business	Technical	Management	Priority Vector	Local Weights
Business	0.391	0.391	0.391	0.391	0.391
Technical	0.304	0.304	0.304	0.304	0.304
Management	0.304	0.304	0.304	0.304	0.304

For instance normalized value for business against business, field 1 of column 1, is $1/2.556=0.391$ and normalized value for technical against business, field 2, column 1, is $0.778/2.556=0.304$. Then averages of normalized values in each row are derived which are the respective priority vector values. Average of row 1 of table 3 which is for business skills criterion is 0.391. Then arithmetic mean for technical and

management criteria are 0.304. To derive the local weights for each criterion, priority vector values are normalized by finding the quotient of each vector value by the sum of the vector values. For example local weight for business criterion is $0.391/(0.391+0.304+0.304)=0.391$.

To determine if the data collected from evaluators were consistent, maximum approximate Eigen value, λ_{max} , is calculated by finding the sum of the products of priority vector values of criterion in table 3 and respective totals of the column of PCM values for the respective criterion in Table 2. In this case $\lambda_{max} = 2.556 \times 0.391 + 3.286 \times 0.304 + 3.286 \times 0.304 = 3.0$. Saaty [13] suggests that Consistency Index (CI) of a matrix of order n is $(\lambda_{max}-n)/(n-1)$ and values are consistent if $CI \leq 0.1$. In this case, $n=3$ and $CI = (3-3)/2 = 0$. This process is repeated for level 3 and 4 to find local weights for sub criteria and partners.

Global weights are derived by merging/multiplying local weights of alternatives at lower levels in the

hierarchy to local weights of alternatives in the parent levels in the hierarchy. The averages of partners' evaluators' opinions after conversion from linguistic to Saaty scale for Business sub criteria; financial security (FS), business strength (BS) and strategic position (SP) were 9, 5, 3 respectively; Technical sub criteria, technical capability (TC), development speed (DS), cost of development (CD) and information technology (IT) were 9, 5, 7 and 3 respectively and Management sub criteria, collaboration record (CR), cultural compatibility (CC) and management ability (MA) were 9, 3 and 5 respectively. For each sub criterion, partners 1 to 5 were evaluated. Table 4 below summarizes the results of this process.

Table 4 Results of Evaluators Data by AHP

Criteria	Local weight	Sub-criteria	Local weight	Global weight	P1	P2	P3	P4	P5
Business	0.391	FS	0.527	0.206	0.333	0.167	0.233	0.112	0.155
		SP	0.170	0.066	0.433	0.167	0.111	0.101	0.188
		BS	0.303	0.118	0.285	0.143	0.333	0.154	0.085
Technical	0.304	TC	0.379	0.115	0.188	0.250	0.167	0.274	0.121
		DS	0.214	0.065	0.129	0.375	0.115	0.122	0.259
		CD	0.286	0.087	0.250	0.150	0.368	0.211	0.021
		IT	0.121	0.037	0.133	0.267	0.267	0.194	0.139
Management	0.304	CR	0.496	0.151	0.367	0.333	0.211	0.022	0.067
		CC	0.188	0.057	0.200	0.100	0.066	0.289	0.345
		MA	0.316	0.096	0.100	0.400	0.315	0.179	0.006
				Priority Weight	0.264	0.233	0.229	0.150	0.122
				Total	0.998				
				Error	0.002				

Global weight (GW) for FS is derived by multiplying local weight of Business criterion by local weight of FS, that is $0.391 \times 0.527 = 0.206$, GW for TC is $0.304 \times 0.379 = 0.115$. Likewise GW for CC is $0.304 \times 0.188 = 0.057$. Finally priority weights (PWs) for partners are derived by finding the sum of products of global weights of each sub criterion and the local weight of the partner in the sub criterion. For instance PW for partner 1 is $0.206 \times 0.333 + 0.066 \times 0.433 + 0.118 \times 0.285 + 0.155 \times 0.188 + 0.065 \times 0.129 + 0.087 \times 0.250 + 0.037 \times 0.133 + 0.151 \times 0.367 + 0.057 \times 0.200 + 0.096 \times 0.100 = 0.264$. PWs for partners 2 to 5 are derived in the same way. If all was perfect the sum of the weights for partners should be 1. From Table 4 the sum is 0.998 with an error of

0.002. The overall weights of Partner 1 through 5 were 0.264, 0.233, 0.229, 0.150 and 0.122 respectively. Partner 1 had the highest weight and was consequently selected.

4.2 Fuzzy Analytical Hierarchy Process

This algorithm introduce fuzzy logic in AHP [20, 16]. The evaluators' judgments are normally vague and difficult to represent in terms of exact precise numbers. It could best be given as interval judgements than fixed value judgements. The process of Fuzzy AHP proposed for this study is shown in Figure 3 below and the following sections.

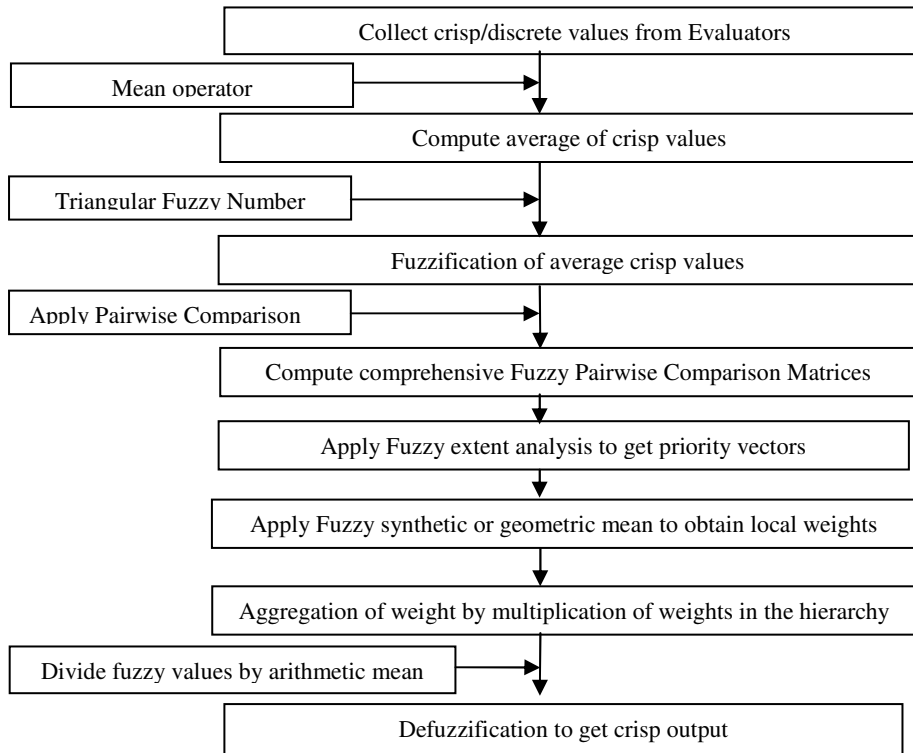


Figure 3 Fuzzy AHP for partner selection and evaluation problem

First, each of the 80 evaluators use the questionnaire (in appendix) to indicate the level of importance of each criteria and sub criteria and their preferences for each partner by assigning crisp values. Second, the crisp values opinions from evaluators are aggregated using arithmetic mean method. Third, the aggregated opinions are converted to triangular fuzzy numbers (TFN). Table 2 and Figure 4 below illustrates the conversions from crisp to fuzzy values and fuzzy

membership function respectively. The outcome of this step is a comprehensive fuzzy opinions. In the study, the aggregated crisp opinions from the crisp values were 9, 7, 7 for business, technical and management skills respectively. These crisp values are converted to fuzzy values. Crisp value of business criterion of 9 was converted to (7, 9, 9). Fuzzy values for Technical and Management criteria were (5, 7, 9) and (5, 7, 9) respectively.

Table 2 Membership function for conversion of crisp to fuzzy values

Crisp number	1	3	5	7	9
Fuzzy Membership function	(1, 1, 3)	(1, 3, 5)	(3, 5, 7)	(5, 7, 9)	(7, 9, 9)

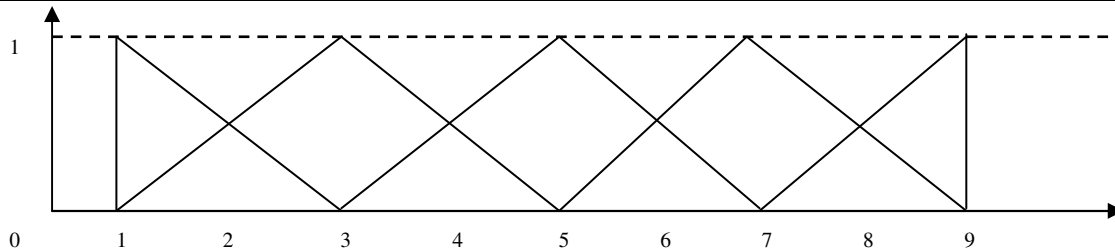


Figure 4 Fuzzy Membership function

Fourth step is the computation of a comprehensive fuzzy pairwise comparison matrices. Fuzzy PCM for

these values is shown in Table 3.

Table 3 Fuzzy Pairwise Comparison for Criteria

Criteria	Business	Technical	Management
Business	1, 1, 1	7/5, 9/7, 9/9	7/5, 9/7, 9/9
Technical	9/9, 7/9, 5/7	1, 1, 1	1, 1, 1
Management	9/9, 7/9, 5/7	1, 1, 1	1, 1, 1
Sum	3, 2.556, 2.428	3.4, 3.286, 3	3.4, 3.286, 3

Values in field 1, column 1 for business against itself is (1,1,1) which is found by dividing lower bound value by lower bound value, middle value by middle value and upper bound value by upper bound value (7/7, 9/9, 9/9). Values in field 3, column 1, is found by dividing (7, 9, 9) by (5, 7, 9). Other field values are derived in the same manner. The sum of each column is found by adding lower bound values together, middle values together and upper bound values together. That is sum of column 1 is (1+1+1=3), (1+7/9+7/9=2.556) and (1+5/7+5/7=2.428). Sums of columns 2 and 3 are found in the same manner.

The fifth step is the derivation of relative local weights of alternatives in each level of the hierarchy by extent analysis of the fuzzy PCM. The basic procedures for fuzzy extent are adopted from [21] are as follows:

Let $X = \{x_1, x_2, x_3 \dots x_n\}$ be an object set (objective, selection criteria, or selection sub-criteria) and $G = \{g_1, g_2, g_3, \dots g_n\}$ be a goal defined for each level in the hierarchical structure. Thus, G can change depending on the level of the hierarchy.

M extent analysis on each object is taken $\hat{M}_{gi}^1, \hat{M}_{gi}^2, \hat{M}_{gi}^3, \dots, \hat{M}_{gi}^m, i=1, 2, 3, \dots, n,$

where \hat{M}_{gi}^j (j=1, 2, 3, ..., m) are TFNs. The fuzzy synthetic extent value (S) with respect to the i^{th} object is defined as,

$$S_i = \sum_{j=1}^m \hat{M}_{gi}^j * [\sum_{i=1}^n \sum_{j=1}^m \hat{M}_{gi}^j]^{-1}.$$

To obtain $\sum_{j=1}^m \hat{M}_{gi}^j$, perform the normalized fuzzy addition operation of m extent analysis values for a particular matrix such that

$$\sum_{j=1}^m \hat{M}_{gi}^j = (\sum_{j=1}^m l_j, \sum_{j=1}^m m_j, \sum_{j=1}^m u_j),$$

where l is the lower limit value, m is the most likely and u is the upper limit value. Table 3 is normalized by dividing each fuzzy number in a column with the respective sum of the column. That is, lower bound elements are divided by the sum of lower bound elements. Likewise the same is done to middle and upper bound elements. Normalization for columns 2 and 3 is done in the same way. Table 4 is the normalized Fuzzy PCM of Table 3.

Table 4 Normalized Fuzzy PCM for Criteria

Criteria	Business	Technical	Management	Fuzzy Addition= $\sum_{j=1}^m \hat{M}_{gi}^j$
Business	0.333, 0.391, 0.412	0.412, 0.391, 0.333	0.412, 0.391, 0.333	1.157, 1.173, 1.078
Technical	0.333, 0.304, 0.294	0.294, 0.304, 0.333	0.294, 0.304, 0.333	0.921, 0.912, 0.960
Management	0.333, 0.304, 0.294	0.294, 0.304, 0.333	0.294, 0.304, 0.333	0.921, 0.912, 0.960
Sum= $\sum_{i=1}^n \sum_{j=1}^m \hat{M}_{gi}^j$				2.999, 2.997, 2.998
Inverse of sum				0.333, 0.334, 0.334

Field 1, column 1 values are derived as (1/3=0.333, 1/2.556=0.391, 1/2.428=0.412).

Fuzzy addition for business criterion, field 1, column 3 is achieved as 0.333+0.412+0.412=1.157, 0.391+0.391+0.391=1.173; 0.412+0.333+0.333=1.078. Other criteria's fuzzy addition is done in a similar manner. The last column of the last row which is the sum of results of normalized PCM fuzzy addition operation of \hat{M}_{gi}^j (j=1, 2, ..., m) values such that $\sum_{i=1}^n \sum_{j=1}^m \hat{M}_{gi}^j = (\sum_{i=1}^n l_i, \sum_{i=1}^n m_i, \sum_{i=1}^n u_i)$

$\sum_{i=1}^n \sum_{j=1}^m \hat{M}_{gi}^j$ in table 4 is computed as follows:

$$1.157+0.921+0.921=2.999;$$

$$1.173+0.912+0.912=2.997;$$

$$1.078+0.960+0.960=2.998$$

To obtain inverse of $\sum_{i=1}^n \sum_{j=1}^m \hat{M}_{gi}^j$ is then computed, such that:

$$[\sum_{i=1}^n \sum_{j=1}^m \hat{M}_{gi}^j]^{-1} = (\frac{1}{\sum_{i=1}^n l_i}, \frac{1}{\sum_{i=1}^n m_i}, \frac{1}{\sum_{i=1}^n u_i})$$

Note: Inverse of a fuzzy number N (l, m, u) is $N^{-1} (1/l, 1/m, 1/u)$

From Table 4, inverses are $1/2.999=0.333$, $1/2.997=0.334$, $1/2.998=0.334$

Extent analysis values are found by multiplying the normalized fuzzy addition of each criteria by the inverse of the sums of the normalized fuzzy addition thus $S_i = \sum_{j=1}^m \hat{M}_{gi}^j * [\sum_{i=1}^n \sum_{j=1}^m \hat{M}_{gi}^j]^{-1}$.

$1.157 \times 0.334, 1.173 \times 0.334, 1.078 \times 0.333 = 0.386, 0.392, 0.359$

$0.921 \times 0.334, 0.912 \times 0.334, 0.960 \times 0.333 = 0.308, 0.305, 0.320$

$0.921 \times 0.334, 0.912 \times 0.334, 0.960 \times 0.333 = 0.308, 0.305, 0.320$

These results are fuzzy. The last step is to find the local weights. For each block, geometric mean of the fuzzy extent values is computed. This gives the priority vector, V_i , for each block. Table 5 show the outcome of this process. The last column of the matrix is determined by finding geometric mean of the fuzzy vectors. Thus, for the first row: $(0.386 \times 0.392 \times 0.359)^{1/3} = 0.379$

Table 5 Local Weight

Criteria	Fuzzy Priority Vector	Defuzzified Priority Vector	Local Weights
Business	0.386, 0.392, 0.359	0.379	0.379
Technical	0.308, 0.305, 0.320	0.311	0.311
Management	0.308, 0.305, 0.320	0.311	0.311

The same procedure is done when finding the priority vectors and local weights for all levels in the hierarchy. Global weights are derived like in AHP.

Table 6 shows the outcome when data from evaluators were subjected to Fuzzy AHP.

Table 6 Results of Evaluators' Data by Fuzzy AHP

Criteria	Local weight	Sub-criteria	Local weight	Global weight	P1	P2	P3	P4	P5
Business criterion	0.379	FS	0.413	0.157	0.333	0.167	0.233	0.112	0.155
		SP	0.303	0.115	0.433	0.167	0.111	0.101	0.188
		BS	0.282	0.107	0.285	0.143	0.333	0.154	0.085
Technical criterion	0.311	TC	0.288	0.090	0.188	0.250	0.167	0.274	0.121
		DS	0.200	0.062	0.129	0.375	0.115	0.122	0.259
		CD	0.140	0.044	0.250	0.150	0.368	0.211	0.021
		IT	0.371	0.115	0.133	0.267	0.267	0.194	0.139
Management criterion	0.311	CR	0.488	0.152	0.367	0.333	0.211	0.022	0.067
		CC	0.280	0.087	0.200	0.100	0.066	0.289	0.345
		MA	0.231	0.072	0.100	0.400	0.315	0.179	0.006
				Priority Weight	0.264	0.231	0.214	0.151	0.140
				Total	1.000				
				Error	0				

Global weight (GW) for SP is derived by multiplying local weight of Business criterion by local weight of SP, that is $0.379 \times 0.303 = 0.115$, GW for CD is $0.311 \times 0.140 = 0.044$. Likewise GW for MA is $0.311 \times 0.231 = 0.072$. Finally priority weights (PWs) for partners is derived by finding the sum of products of global weights of each sub criterion and the local weight of the partner in the sub criterion. For instance

PW for partner 2 is $0.157 \times 0.167 + 0.115 \times 0.167 + 0.107 \times 0.143 + 0.090 \times 0.250 + 0.062 \times 0.375 + 0.044 \times 0.150 + 0.115 \times 0.267 + 0.152 \times 0.333 + 0.087 \times 0.100 + 0.072 \times 0.400 = 0.231$. PWs for partners 1, 3 to 5 are derived in the same way. If all was perfect the sum of the weights for partners should be 1. From table 6 the sum is 1.0 with an error of 0. The overall weights of Partner 1 through 5 were

0.264, 0.231, 0.214, 0.151 and 0.140 respectively. Partner 1 had the highest weight and was consequently selected.

4.3 Reduced Group Fuzzy AHP

This is a new algorithm that has both features for AHP and Fuzzy AHP. First, the decision makers give their evaluation comparison judgements of different partners in crisp values, as it is done in AHP. The crisp values from evaluators are fuzzified using triangular fuzzy number as it is done in FAHP. The average of the fuzzified evaluators' opinions is

computed and a fuzzy pairwise comparison matrices are formed. The fuzzy comparison matrices are split into two parts. The lower bound values are used to form lower PCMs while upper bound values form upper PCMs. These PCMs have crisp values, therefore, AHP approach is used to derive priority vectors and local weights after confirming the evaluators' consistency using the method in [13]. Local weights of alternatives in lower PCM is combined with local of alternatives in upper PCM using geometric mean. Figure 5 and subsequent sections describe Reduced Group FAHP.

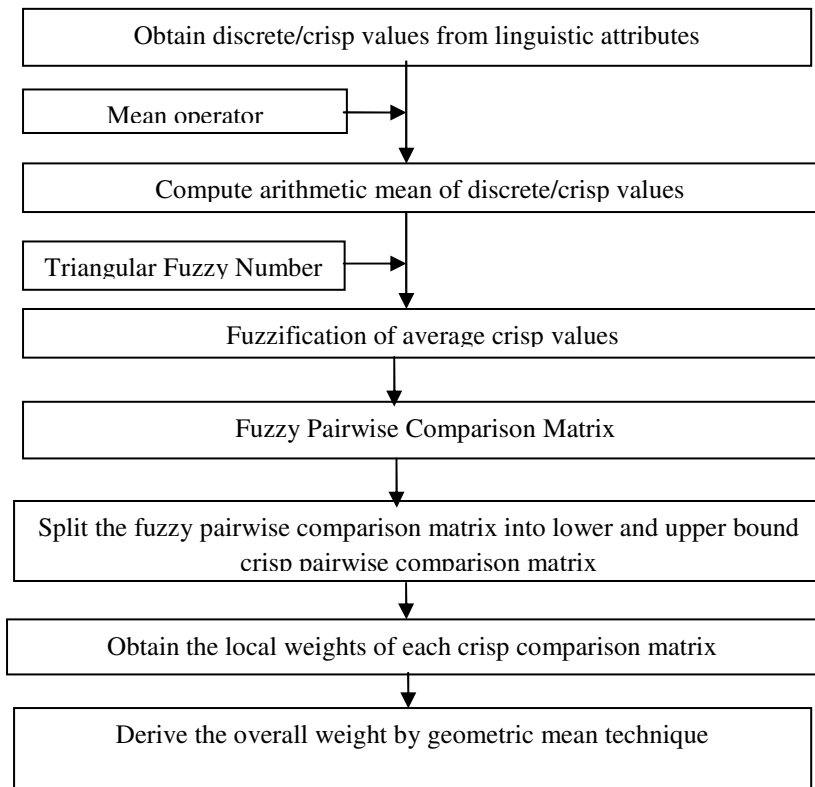


Figure 5 Reduced Group Fuzzy AHP

First, each of the evaluators use the questionnaire (in appendix) to indicate the level of importance of criteria and sub criteria and their preferences for each partner by assigning crisp values. Second, the

arithmetic mean of crisp evaluators' opinion values is computed. Third, the aggregated crisp values are converted to triangular fuzzy numbers (TFN). Table 2 and Figure 4 in section 4.2 illustrate the conversions from crisp to fuzzy values and fuzzy membership function respectively. Fourth, compute a comprehensive PCM. Fuzzy PCM for these values is shown in Table 7.

Table 7 Fuzzy Pairwise Comparison for Criteria

Criteria	Business	Technical	Management
Business	1, 1, 3	7/5, 9/7, 9/9	7/5, 9/7, 9/9
Technical		1, 1, 3	1, 1, 3
Management			1, 1, 3

Fifth, the fuzzy pairwise comparison matrix is divided into two matrices consisting of lower and upper bound elements as shown in Tables 8 and 9.

Table 8 Lower PCM for Criteria

Criteria	Business	Technical	Management
Business	1.00	1.40	1.40
Technical	0.714	1.00	1.00
Management	0.714	1.00	1.00

The elements of the lower diagonal of the pairwise comparison matrix is filled by computing the reciprocal of each corresponding element.

Table 9 Upper PCM for Criteria

Criteria	Business	Technical	Management
Business	3.00	1.00	1.00
Technical	1.00	3.00	3.00
Management	1.00	0.33	3.00

Using AHP approach local weights for lower and upper bound elements is derived as shown in Table 10.

Table 10 Priority weights for Criteria

Criteria	Lower local Weight	Upper local Weight	Overall Weight (Geometric Mean)
Business	0.412	0.325	0.366
Technical	0.294	0.441	0.360
Management	0.294	0.235	0.263

After obtaining the results for the local weights of the lower and upper elements then the final step is to combine two respective overall local weights (for the lower and upper element) in order to get the overall

weights for alternatives. This process was applied to values in all levels of the hierarchy and results are shown in Table 11. Global weight (GW) for BS is derived by multiplying local weight of Business

criterion by local weight of BS, that is $0.366 \times 0.176 = 0.064$, GW for DS is $0.360 \times 0.211=0.076$. Likewise GW for CR is $0.263 \times 0.499=0.118$. Finally PWs for partners is derived by finding the sum of products of global weights of each sub criterion and the local weight of the partner in the sub criterion. For instance PW for partner 1 is

$$0.191 \times 0.333 + 0.111 \times 0.433 + 0.064 \times 0.285 + 0.112 \times 0.118 + 0.076 \times 0.129 + 0.045 \times 0.250 +$$

$$0.126 \times 0.133 + 0.118 \times 0.367 + 0.078 \times 0.200 + 0.067 \times 0.100 = 0.254.$$

PWs for partners 1, 3 to 5 are derived in the same way. If all was perfect the sum of the weights for partners should be 1. From table 11 the sum is 0.987 with an error of 0.013. The overall weights of Partner 1 through 5 was 0.254, 0.230, 0.207, 0.153 and 0.143 respectively. Partner 1 had the highest weight value and was consequently selected.

Table 11 Results of Evaluators' Data by Reduced Group Fuzzy AHP

Criteria	Local weight	Sub-criteria	Local weight	Global weight	P1	P2	P3	P4	P5
Business Skills Cluster	0.366	FS	0.521	0.191	0.333	0.167	0.233	0.112	0.155
		SP	0.303	0.111	0.433	0.167	0.111	0.101	0.188
		BS	0.176	0.064	0.285	0.143	0.333	0.154	0.085
Technical Skills Cluster	0.360	TC	0.312	0.112	0.188	0.250	0.167	0.274	0.121
		DS	0.211	0.076	0.129	0.375	0.115	0.122	0.259
		CD	0.126	0.045	0.250	0.150	0.368	0.211	0.021
		IT	0.351	0.126	0.133	0.267	0.267	0.194	0.139
Management Skills Cluster	0.263	CR	0.449	0.118	0.367	0.333	0.211	0.022	0.067
		CC	0.298	0.078	0.200	0.100	0.066	0.289	0.345
		MA	0.254	0.067	0.100	0.400	0.315	0.179	0.006
				Priority	0.254	0.230	0.207	0.153	0.143
				Total	0.987				
				Error	0.013				

5. COMPARISON OF THE THREE ALGORITHMS

This section analyses of outcomes of AHP, Fuzzy AHP and Reduced Group Fuzzy AHP algorithms, when the same data set is used. Table 12 shows the outcomes of the three algorithms.

Table 12 Comparison of Outcomes of three algorithms

Method	P1	P2	P3	P4	P5	Total	Error
Conventional AHP (crisp values)	0.264	0.233	0.229	0.150	0.122	0.998	0.002
Fuzzy AHP (fuzzy values)	0.264	0.231	0.214	0.151	0.140	1.00	0
Reduced Group Fuzzy AHP (fuzzy values)	0.254	0.230	0.206	0.153	0.143	0.987	0.013

To verify these results, five case studies were conducted. Evaluators from the cases gave their opinions about the five partners using the questionnaire by examining the partners' company profiles. Averages of the outcomes were computed

and their average errors are shown in Table 13. From these comparisons, it can be deduced that Fuzzy AHP is the most accurate with a mean error of 0.0032 followed by conventional AHP with a mean error of 0.0064 and Reduced Group Fuzzy AHP which has a

mean error of 0.0134. Since the consistency ratio correlate to the judgemental errors in pairwise

comparisons [22], it can be concluded that these mean errors correspond to the consistency ratio [13].

Table 13 Arithmetic Mean Total and Error of Three Algorithms Comparison

Method	Case 1	Case 2	Case 3	Case 4	Case 5	Total	Mean Total	Mean Error
Conventional AHP (crisp values)	0.997	0.989	0.998	0.996	0.988	4.968	0.9936	0.0064
Fuzzy AHP (fuzzy values)	0.996	0.995	0.997	1	0.996	4.984	0.9968	0.0032
Reduced Group Fuzzy AHP (fuzzy values)	0.988	0.981	0.986	0.99	0.988	4.933	0.9866	0.0134

6. FUZZY INFERENCE SYSTEM

Fuzzy inference is a method of interpreting values in the input vector and assigning values to the output vector based on a set of rules. A Fuzzy inference system (FIS) can be used to aid decision making [23]. A FIS is built based on the idea of fuzzy sets and fuzzy numbers. A fuzzy set is a set without a crisp, clearly defined boundary. Fuzzy numbers represent a number whose value is somewhat uncertain. It was suggested that triangular fuzzy numbers are appropriate for quantifying the vague information about most decision problems including personnel selection due to their simplicity and their intuitive and computational-efficient representation [24]. Fuzzy inference process comprised the following steps: fuzzification of the input variables, application of the fuzzy operator in the antecedent, implication from the antecedent to the consequent, aggregation of the consequents across the rules, and defuzzification.

5.1 Formulating the FIS for PESP

In literature, there are two basic approaches of fuzzy system modelling, i.e. linguistic fuzzy modelling and precise fuzzy modelling [25]. Linguistic fuzzy modelling, also known as the Mamdani approach, has high interpretability but lacks accuracy. On the other hand, precise fuzzy modelling, such as the Sugeno-type fuzzy inference, exhibits high accuracy but at the cost of interpretability. The accuracy of a fuzzy model indicates how closely it can represent the system, while interpretability is a measure of understanding of the system behaviour and expressing it through the model. Mamdani FIS, unlike Sugeno-type FIS, requires only a small input-output database for tuning and can interpret system behaviour between the discrete data. It is more intuitive and suited to human input. This study use a Mamdani inference engine for the proposed fuzzy model for PESP.

In the present problem of determining the best partner for a given construction task, input variables

used are the overall weights of importance of business, technical and management skills while the output is the ranking of scores which can be used for decision making. During fuzzification, the antecedent variables of the system are converted into fuzzy variables using fuzzy sets. As discussed earlier, fuzzy sets associate a membership function (denoted $\mu(x)$) which maps an input value to its appropriate membership value. Membership function can be an arbitrary function with values in (0,1).

Triangular membership functions (MFs) were chosen to describe the fuzziness of input and output variables. There are two stages to the PESP procedure. The first is data preparation and processing, and the second is fuzzy inference using the designed FIS to obtain the final evaluation score. The first stage entails the use of FAHP data collection and weighting techniques. Stage two entails FIS application formulation for PESP which takes in three inputs in the form of overall weights of importance for business, technical and management skills criteria and uses Mamdani-type fuzzy inference to produce an output evaluation score. This score can then be used to rank the evaluated partners to aid the decision-making process.

7. TIME COMPLEXITIES

Time complexity refers to time in which the algorithm runs. It is determined by finding the upper bound on the execution time [26]. In AHP, the computational time is affected by the size of a matrix, with bigger matrices requiring more time [27]. Considering a prioritization of n elements stated as T_1, T_2, \dots, T_n , the intensity of preference element T_i over element T_j which represent a judgment is indicated as a_{ij} for $i, j = 1, 2, \dots, n$ [28]. If element T_i is preferred to T_j , then $a_{ij} > 1$ or otherwise $a_{ij} < 1$ and $a_{ij} = 1$ (for all $i, j = 1, 2, \dots, n$) when the two elements is of the same importance. Hence, the reciprocal property $a_{ji} = 1/a_{ij}$ by assumption will always hold, with $a_{ii} = 1$

(for all $i= 1, 2,\dots, n$) [29,30]. Finally, a positive reciprocal matrix of pairwise comparison with the property $A=a_{ij}$ is constructed by having a dimension of $n \times n$ [27].

Consider an AHP reciprocal matrix A with weights,

$$A = \begin{matrix} & T_1 & T_2 & \dots\dots & T_n \\ \begin{matrix} T_1 \\ T_2 \\ \dots \\ T_n \end{matrix} & \begin{pmatrix} a_{11} & a_{12} & \dots\dots & a_{1n} \\ a_{21} & a_{22} & \dots\dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots\dots & a_{nn} \end{pmatrix} & \text{Weights} = \begin{pmatrix} W_1 \\ W_2 \\ \dots \\ W_n \end{pmatrix} \end{matrix}$$

where n is the number of elements and T are the objects while W is the derived weights from the reciprocal matrix. For the elements of the main diagonal in matrix A which are a_{11}, \dots, a_{nn} , the elements will always be equal to 1. Due to the reciprocal nature of AHP matrix, judgments are only required to the upper diagonal of the matrix and only need $n(n-1)/2$ of the judgments to generate a matrix for prioritization while the symmetrical elements are communally reciprocal [29]. This means that the elements below the diagonal elements are satisfying the equation which is $a_{ji} = 1/a_{ij}$.

If there are n selection criteria and m candidates, the evaluators would have to make $n(n-1)/2 + n(m(m-1)/2)$ pairwise comparisons, a substantial number even for a small n and m (<8). Chang [26] found FAHP (for n criteria) has the time complexity of $n(n+6)$ and AHP has a time complexity equal to $\frac{n(n-1)}{2}$. The number of comparisons in RGFAHP is twice that of AHP. This is due to the fact that once linguistic evaluations are converted to fuzzy values, two matrices are formed. One matrix is formed using lower bound elements and the other matrix is formed using the upper bound elements. Pairwise comparisons for each matrix is computed using AHP approach. One matrix of n criteria will take $p = \frac{n(n-1)}{2}$ comparisons. For the two matrices, the number of comparisons is twice p comparisons. $2 \times \frac{n(n-1)}{2} = n(n-1)$. Using comparisons RGFAHP has a time complexity of $n(n-1)$.

8. DISCUSSIONS

All the three algorithms have errors. It is important to note that AHP cannot be used in situation which has fuzziness (uncertainty). However, FAHP and RGFAHP algorithms are better because they can be used when the evaluators judgement are both certain and uncertain. This is due to the fact that both FAHP and RGFAHP are based on the principles of AHP. The three algorithms are effective but FAHP and RGFAHP outweigh AHP in terms of generality. This is because FAHP and RGFAHP can be used when evaluator judgements are either exact or fuzzy. RGFAHP outweigh FAHP because it has fewer steps. In addition RGFAHP has characteristics of both AHP and FAHP. In terms of accuracy, AHP has a mean accuracy of 99.32%, FAHP has a mean accuracy of 99.68% while RGFAHP has a mean accuracy of 98.66%. Apart from the correctness, simplicity and generality of the algorithm, other aspects which can be used to differentiate between the algorithms are time and space complexities. Time complexity refers to time in which the algorithm runs. It is determined by finding the upper bound on the execution time [26]. Chang [26] found FAHP (for n criteria) has the time complexity of $n(n+6)$ and AHP has a time complexity equal to $\frac{n(n-1)}{2}$. RGFAHP has a time complexity between that of AHP and FAHP but twice that of AHP which is $n(n-1)$. AHP algorithm can be extended to be used in a situation where the evaluator has imprecise information about evaluation judgements. Fuzzy logic can be incorporated in AHP to address the uncertainty of user judgement during the evaluation of partners. These algorithms gave approximately similar results.

8.1 Conclusions and further work

Researchers should consider how the results of this study can be used for partner evaluation and selection problems in general. More research should be carried out to determine the applicability of AHP, FAHP and RGFAHP to other industries and other research fields. The limitations of FAHP and RGFAHP should probably be addressed in future research. Examples of limitations are: (i) checking if FAHP and RGFAHP preserve the consistency of the evaluator's judgement; and (ii) whether FAHP and RGFAHP ignore the dependence between the elements at the same level of the hierarchy, as is the case with AHP.

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APPENDIX: QUESTIONNAIRE

Collaboration of Enterprises

Indicate your choice with a tick (√) on the label provided. For the purpose of this study the term “collaboration” is defined as participation in a project between organizations that operate under a different management.

Section A-Partners Selection and Evaluation Criteria

1. Indicate how important each of the following criterion is when your company is selecting partners for a task in a building construction project. Use the symbols “A to E” with A being “Extremely important” and E being “Not at all important”. Choose the symbol which best indicates your choice.					
Criterion	Extremely	Very important	Important	Weakly	Not at all

		important			important	important
Business Skills		A	B	C	D	E
Technical Skills		A	B	C	D	E
Management Skills		A	B	C	D	E
2. Considering Business Skills Criterion; indicate how important each of the following sub-criteria is when your company is selecting partners for a task in a building construction project. Use the symbols "A to E" with A being "Extremely important" and E being "Not at all important". Choose the symbol which best indicates your choice.						
Sub-Criteria		Extremely important	Very important	Important	Weakly important	Not at all important
Business Strength (BS)		A	B	C	D	E
Financial Security (FS)		A	B	C	D	E
Strategic Position (SP)		A	B	C	D	E
3. Considering Technical Skills Criterion; indicate how important each of the following sub-criteria is when your company is selecting partners for a task in a building construction project. Use the symbols "A to E" with A being "Extremely important" and E being "Not at all important". Choose the symbol which best indicates your choice.						
Sub-Criteria		Extremely important	Very important	Important	Weakly important	Not at all important
Technical Capabilities (TC)		A	B	C	D	E
Development Speed (DS)		A	B	C	D	E
Cost of Development (CD)		A	B	C	D	E
Information Technology (IT)		A	B	C	D	E
4. Considering Management Skills Criterion; indicate how important each of the following sub-criteria is when your company is selecting partners for a task in a building construction project. Use the symbols "A to E" with A being "Extremely important" and E being "Not at all important". Choose the symbol which best indicates your choice.						
Sub-Criteria		Extremely important	Very important	Important	Weakly important	Not at all important
Collaboration Record (CR)		A	B	C	D	E
Cultural Compatibility (CC)		A	B	C	D	E
Management Ability (MA)		A	B	C	D	E

Section B-Partner Selection

Use the company profiles of companies P1, P2,...P5 provided at the end of this questionnaire. Indicate how preferable is each company against each other according to partner selection sub-criterion to perform a task in a building construction project. Use the symbols "A to E" with A being "Extremely preferable" and E being "Not at all preferable". Choose the symbol which best indicates your choice.						
Sub-Criteria		Extremely preferable	Strongly preferable	Preferable	Weakly preferable	Not at all preferable
		P1 P2 P3 P4 P5	P1 P2 P3 P4 P5	P1 P2 P3 P4 P5	P1 P2 P3 P4 P5	P1 P2 P3 P4 P5
Technical capabilities (Have relevant types of skills)		A A A A A	B B B B B	C C C C C	D D D D D	E E E E E
Development speed (Can complete tasks within project timelines)		A A A A A	B B B B B	C C C C C	D D D D D	E E E E E
Financial security (Amount of money deposited before project commencement)		A A A A A	B B B B B	C C C C C	D D D D D	E E E E E
Collaborative record (Have been part of large projects)		A A A A A	B B B B B	C C C C C	D D D D D	E E E E E
Business strength (Have necessary equipment and qualified staff)		A A A A A	B B B B B	C C C C C	D D D D D	E E E E E
Cost of development (The projected task cost within the project budget)		A A A A A	B B B B B	C C C C C	D D D D D	E E E E E
Corporate cultural compatibility (Staff management style in the previous projects)		A A A A A	B B B B B	C C C C C	D D D D D	E E E E E
Strategic position (Partnership with other firms like financiers)		A A A A A	B B B B B	C C C C C	D D D D D	E E E E E
Management ability (Handles staff issues amicably)		A A A A A	B B B B B	C C C C C	D D D D D	E E E E E
Use of Information Technology (Use software for designs, finance and staff issues management)		A A A A A	B B B B B	C C C C C	D D D D D	E E E E E