

Bandits using Fuzzy to fill Knapsacks: Smallholder Farmers Credit Scoring

Benjamin OTIENO¹, Franklin WABWOBA¹, George MUSUMBA²

¹Kibabii University, P.O. Box 1699, Bungoma, 50200, Kenya

Tel: +254 733365384, Email: otienobenedjamin@gmail.com, fwabwoba@kibu.ac.ke

²Dedan Kimathi University of Technology, Private Bag-10143, Nyeri, Kenya

Email: musumbaggw@gmail.com

Abstract: Consumer lending is an exercise where risk scoring takes the form of a typical decision making problem. For smallholder farmers, the credit scoring becomes specifically more challenging with the data gaps and outliers in data. Added to that, the process must be as cost-effective as possible while providing as accurate results as possible. This paper uses data obtained from smallholder farmers in Kakamega County in Kenya to set up an experiment of credit scoring as a Bandit with Knapsack problem with Fuzzy Unordered Rule Induction Algorithm (FURIA) being used as the exploit-explore algorithm and Fuzzy Analytical Hierarchical Process (FAHP) used to determine the ranking and consistency of the FURIA rules. The experiment returns a consistency ratio of 0.000529 which is significantly less than the 0.10 threshold. In this regard, the paper proposes the use of FURIA to reduce the regret in Bandit with Knapsack (BwK) as a technique for smallholder credit scoring.

Keywords: smallholder farmers, risk scoring, FURIA, bandit with knapsack, FAHP

1. Introduction

Lenders usually lend to borrowers, having evaluated their risk. For some class of borrowers, the lenders do individual risk scoring by evaluating certain criteria. This individual scoring can be at times expensive and as such, for the mass market, automated credit scoring mechanisms are often employed. These mechanisms can at times return biased results. Smallholder farmers fall in the category of mass market. However, they have one uniqueness: the data sourced from smallholder farmers is usually full of missing data and outliers. This makes lenders resort to index based risk scoring, bundling all smallholder farmers into one risk basket thus generating bias. This made a case for investigating an automated risk scoring mechanism that can return as close risk scores as those that would be generated by human evaluators; a mechanism that can actively use the missing data and outliers and inspire lender confidence in the resultant scores. The goal is to have a mechanism that does not disadvantage the borrowers on the account of them being smallholder farmers.

In a typical Bandit with Knapsack (BwK) problem, fixed limited set of resources have to be allocated to competing choices in a way to maximize expected gain where limited information on the choices is available. In BwK problems, the dilemma is balancing between exploration and exploitation. BwK used in statistics and machine learning [1] finds application in real-time strategy games [2], probabilistic maximum coverage, social influence and linear reward problems [1]. Practical application areas include healthcare, finance, dynamic pricing recommender systems, influence maximization, information retrieval, dialogue systems, anomaly detection and telecommunication.

In this paper we consider the application of BwK in credit scoring for smallholder farmers. At the inception, a lender has lending headroom and an unknown growing number of loan-seekers. The lender does not know an optimal way to pick the loan-seekers to have the lending headroom completely used while at the same time minimizing the probability of default. And since the list of loan-seekers is continuously growing, the lender does not know whether the best quality of borrowers have already applied for loans or are yet to apply for loans. The expected default probability of the choice is only partially known at the point of deciding whether to lend or not. As the borrower starts servicing the loan, more information starts getting available through either good servicing of the loan or default. This poses a typical exploration-exploitation dilemma. The problem becomes more pronounced in smallholder farmers at the hands of lenders as the data is typically incomplete and occasionally with outliers, leading to the bandits regarding the farmers unfit for their knapsacks.

In addressing the exploration-exploitation dilemma, we consider the most optimal dataset that the bandits can use in filling the knapsacks and propose an algorithm for filling the knapsacks. The research uses data from smallholder farmers and financial institutions to determine what traditional data and what new data can be used for this cause without prejudicing the smallholder farmers during credit scoring.

We summarize the main contributions of this paper as follows:

This paper is the first to explore the exploration-exploitation trade-offs involving fuzzy sets and fuzzy events in the context of a credit scoring problem.

Experimental results show that the consistency ratio for the fuzzy analytical hierarchical process is within acceptable levels with the potential of reducing the regret thus making fuzzified BwK fit for credit scoring for smallholder farmers.

The rest of the paper is organized with section 2 having works related to the current study, section 3 outlines the problem formulation while section 4 details the methodology. Section 5 presents the results, section 6 has a brief on the technology use case and section 7 outlines the conclusion and future directions.

2. Related literature

Smallholder farmer credit scoring problem has certain uniqueness that make it a subject of interest. The level of bias suffered by the smallholder farmers when lenders use conventional credit scoring algorithms make it important to consider alternative methods [3]. Automated credit scoring for the mass market has its challenges – key of which is the subjective discrimination [4]. There have been extensive studies on classifiers used in consumer credit scoring detailing the upside and downside of each [5] but none of them addresses the peculiar issues specific to smallholder farmers. Some of the unique issues with credit scoring for smallholder farmers include the data gaps and outliers [3] which are more pronounced in this class. In this paper we explore one algorithm that could best fit the smallholder uniqueness.

Credit scoring is a decision-making problem with uncertainty of not knowing the precise credit risk of a loan-seeker upfront [6]. As such, the exercise is made up of a combination of exploring to establish the possible outcomes of credit decisions and exploiting by the actual issuance of credit. Bandits with Knapsacks (BwK) is a decision making model with a balance of trade-offs between exploration and exploitation [7]. BwK have been applied in various application areas such as online learning [8], dynamic pricing and auctions [9] among others. However, there has been no known application of BwK in the credit scoring field. In the current work, we demonstrate how the explore-exploit algorithm can be constructed in the line of BwK for credit scoring. In a paper comparing the fairness and accuracy of various known algorithms, Fuzzy Unordered Rule Induction

Algorithm (FURIA) was found to return better results [3] compared to the others and as such was used in this study as the criteria for choosing the arm in the BwK problem.

The analytical Hierarchical Process (AHP) was introduced by Saaty to solve multi-criteria decision making (MCDM) problem in management [10]. Extension of AHP to the use of Fuzzy AHP (FAHP) has been done by many researchers in different fields [11] [12] [13]. It is considered as a good measure of ranking and consistency of alternatives [14] thus the reason for our choice in using it to rank our Credit Risk Scoring (CRS) criteria. One of the goals is to ensure that the dataset returns desirably low consistency ratios from the FAHP to be used in FURIA rules.

3. Problem formulation

Take a case of a single financial institution (lender) with lending headroom $B = 6,000$ and three smallholder farmers (borrowers) interacting T times with each borrowing cycle capped at 100. Assume that the three borrowers have different risk rating, for simplicity purpose rated as low, medium and high. Given the different default probability of the borrowers, assume the reward differs per borrower as per table 1. Reward here is used to refer to the utility function, taking into consideration the possible loss from default and the interest income.

Table 1: Risk – reward balance of lending

| | Risk | Reward |
|---|--------|--------|
| X | Low | 10% |
| Y | Medium | 8% |
| X | High | 5% |

The challenge is that the lender can only see the reward for the lending decisions made. It is impossible to see what could have happened if a different choice was made.

The other challenge is that the lender does not get all the borrowers coming at the same time, and as such may have to make worse lending decisions where better quality borrowers are yet to come.

The lender's goal is to maximize the revenue by minimizing the default risk. The resource (lending headroom) is in this case limited to B and no more. Once the lender is out of the lending headroom, the lending stops.

The lender has an option of using any one of the three strategies: 1) explore only, 2) exploit only and 3) ϵ -greedy strategy. In each case, we calculate the regret (the reward foregone by the lender) in the simplest manner: total possible reward – actual reward. This makes the regret to be in the order of $O(T)$. In adversarial situation, the regret would be in the order of $O(\log T)$ [15].

In the explore only strategy, the lender will explore each borrower in the same proportion and lend 2,000 to each of the three borrowers. This will result in total reward of

$$\text{Reward} = 10\% \text{ of } 2,000 + 8\% \text{ of } 2,000 + 5\% \text{ of } 2,000$$

$$\text{Reward} = 460$$

Regret is evaluated as the difference between the total reward obtained and the reward that would have been obtained had the player played the arms with the highest reward.

In this case, the highest reward would have been 10% of 6,000 which is 600

The regret in the explore only strategy then becomes $600 - 460 = 140$

The second strategy is the exploit only strategy. In exploit only strategy, the lender explores the three borrowers once, and subsequently lends only to the borrower that returned the greatest reward in the first round. The total reward in this case then evaluates to:

$$\text{Reward} = 10\% \text{ of } 100 + 8\% \text{ of } 100 + 5\% \text{ of } 100 + 10\% \text{ of } 5,700$$

$$\text{Reward} = 593$$

And then the regret evaluates to $600 - 593 = 7$.

Obviously, the exploit only strategy turns out to be better than the explore only as it returns a lower regret (7 compared to 140). However, the exploit only strategy has one major flaw that could lead to fairly high regret. Credit risk is a dynamic variable that could result to default in borrower X on the first loan and consistent high returns in subsequent borrowings. Suppose in the test phase, borrower X returned 7% instead of the 10%, the lender would then lend the 5,700 to borrower Y leading to total reward of 479 and a regret of 121. It is still higher than the explore only but the flaw is not something a lender would want to live with.

The third BwK strategy is ϵ -greedy strategy. Reinforced learning evaluates actions taken rather than instructing with correct actions. The first two strategies are purely reinforced learning strategies. The ϵ -greedy strategy is an improvement that can instruct subsequent choices using past correctness of choices. This strategy can opt for say 30% exploration and 70% exploitation, giving a balance between exploration and exploitation. This paper focuses on this strategy with the use of fuzzy sets and fuzzy events in further reducing the regret in the BwK problem.

4. Methodology

The experiment used data from 49 smallholder farmers from Kakamega County in Kenya and 15 financial institutions having operations in Kakamega County. The data on the importance of the various environmental, farming and financial aspects as used in credit scoring is ranked on importance both by the smallholder farmers and financial institutions. The study ranks the importance of the various CRS data and uses them in a BwK problem with FURIA being used as the explore and exploit algorithm in the BwK problem. From the data used, the researchers determine the consistency ratio for the Fuzzy Analytical Hierarchical Process (FAHP) to determine if the output from FURIA will give the lowest regret in the BwK problem. The experiment proposes an iterative process for the lender with explore and exploit iterations in the order presented in algorithm 1.

Algorithm 1: BwK algorithm

For rounds $t = 1$ to T

Choose an arm $a_t \in K$

Obtain reward $r_t(a_t) \in [0, 1]$

Consume (issue loan) $c_{1,t}(a_t), c_{2,t}(a_t), \dots, c_{d,t}(a_t) \in [0, 1]^d$

Stop if headroom B is exhausted

5. Results

23 questions commonly asked by lenders in CRS were put to both the smallholder farmers and the lenders and each individual in the two groups ranked the criteria on the importance of the questions/criteria (ranging from 1 – 5) Fig 3. The goal was to determine which of the criteria are actually important. The 23 criteria were: Age, Gender, Literacy level, Primary bank, Average bank transactions per month, Marital status, Number of dependants, Number of children, Crops, Land title, Car log book, Guarantor, Financial records, Farming history records, Bank statements, Inputs required per acre, Guarantee of market for produce, Land acreage, Average Rainfall (mm), Average Temperature (degrees Celsius), Source of income, Annual Expenditure (Kenya Shillings) and Insurance.

From the responses, an average of the response from the smallholder farmers for each criterion were done and a similar average for lenders was made. Collecting all the data for CRS can be expensive and as such the goal was to arrive at only the most important data to be collected. The criteria which had convergence in agreement on the level of importance both from the lender and smallholder farmer perspective was picked from the 23 and these were 14. The 14 were: Age, Crop(s), Guarantor, Financial records, farming history records,

Bank statements, Inputs required, Market, Acreage, Average Rainfall, Average Temp, Source of income, Annual Expenditure and Insurance.

For these 14 criteria, both the lender and the smallholder farmers gave a score above 2.5 (out of a total score of 5) indicating that they were in agreement that the criteria were important to risk scoring (Table 2). All criteria that had either the lender or borrower scoring below 2.5 were regarded as not so important and these were trimmed off. For example, Gender was rated 2.7143 by the borrowers and 1.7347 by the lenders hence dropped indicating that the gender question was unimportant to the lenders. Perhaps this could have been because smallholder farming is predominantly done by once gender and as such they had some bias against the other gender, which the lenders did not agree to. Similarly, land title was rated 3.3571 by the lenders while the borrowers rated it at 1.8750 hence being dropped. Perhaps this speaks to the uniqueness of smallholder farmers, many who do not necessarily own the land on which they till versus the traditional manner in which lenders look at borrowers with high regard of land title as security.

The average for both the smallholder farmer and the lender for each criterion was calculated and taken as the zero-day score for the criteria. These are labelled 'Rating' in table 2. We say zero-day scores since the scores are bound to change dynamically for each borrower and these are only to be used in arriving at the CRS model. The ratings were then converted to the 9-point Saaty scale, then further converted to fuzzy scale using the fuzzy numbers in Figure 2.

Table 2: CRS criteria

| Criteria | Age | Crop(s) | Guaran tor | Financial records | Farming history records | Bank statements | Inputs required per acre (Kenya Shillings) | Produce markets (direct, middleman, processor) | Land acreage | Average Rainfall (mm) | Average Temp (degrees Celsius) | Sourc e of inco me | Annu al Expe nd iture (Kenya Shilli ngs) | Insura nce |
|-----------------------|--------|---------|------------|-------------------|-------------------------|-----------------|--|--|--------------|-----------------------|--------------------------------|--------------------|--|------------|
| Importance (borrower) | 2.9796 | 2.7959 | 3 | 2.875 | 3.0408 | 2.7143 | 3.898 | 2.9184 | 3.6327 | 3.8958 | 3.0204 | 3.3469 | 3 | 2.6735 |
| Importance (lender) | 3.3571 | 3.6429 | 3.5 | 3.5714 | 3.9286 | 3.5714 | 3.7143 | 3.7857 | 3.7857 | 3.1429 | 2.7143 | 3.1429 | 3.3571 | 3.5714 |
| Rating | 3.17 | 3.22 | 3.25 | 3.22 | 3.48 | 3.14 | 3.81 | 3.35 | 3.71 | 3.52 | 2.87 | 3.24 | 3.18 | 3.12 |
| Crisp Saaty scale | 3 | 5 | 5 | 5 | 7 | 3 | 9 | 5 | 9 | 7 | 1 | 5 | 3 | 3 |
| Fuzzy Saaty Scale | 2,3,4 | 4,5,6 | 4,5,6 | 4,5,6 | 6,7,8 | 2,3,4 | 8,9,9 | 4,5,6 | 8,9,9 | 6,7,8 | 1,1,2 | 4,5,6 | 2,3,4 | 2,3,4 |

The BwK for the CRS problem is represented by Fig 1. In this model, each lender (bandit) chooses suitable borrowers (smallholder farmers) using explore and exploit algorithm (in this case FURIA) to fill the knapsack (bank assets) with capacity B (lending headroom).

i. Linear program

The goal here is to determine whether the linear program has an upper bound or not. This is because lending as a BwK problem is bound on the upper side.

Using the expected values, we construct a linear function with the function being made up of the summation of the expected outcomes. The function is bounded on the upper side by T for stochastic linear function and the goal is to maximize the reward in time T.

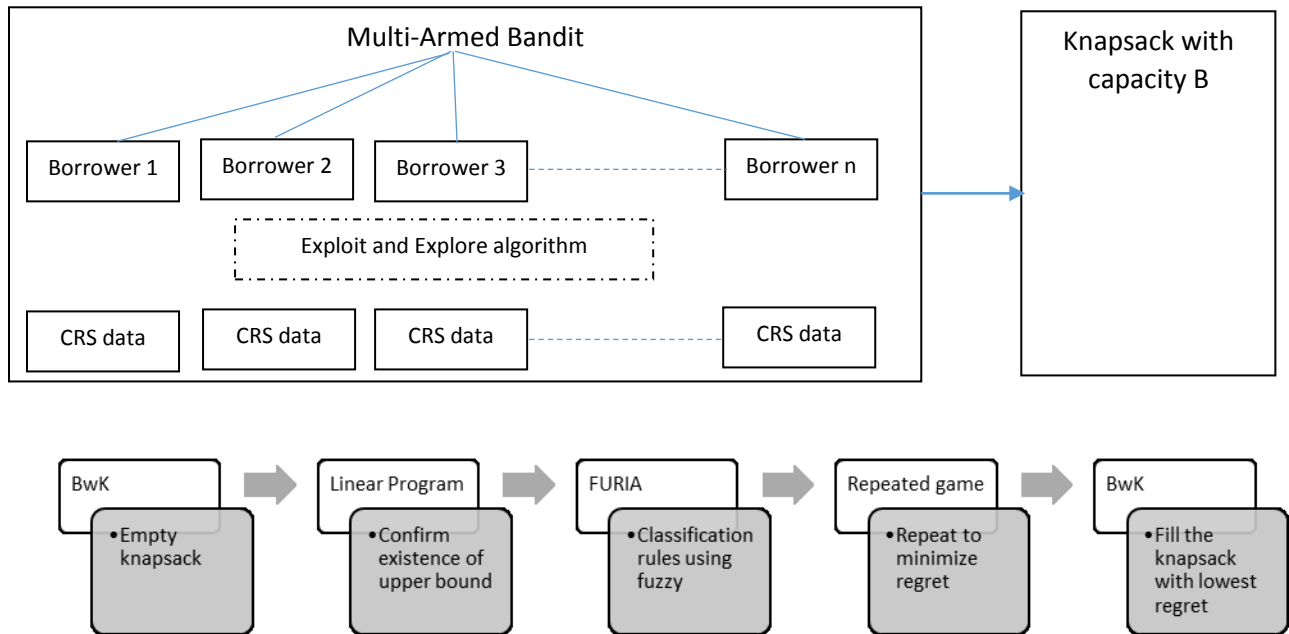


Figure 2: Proposed BwK model for CRS

ii. FURIA

As a departure from the common BwK problems where there is one condition determining the choice of the arm, CRS uses multiple rule set in our case 14. This multiplicity of rules informed the departure from the use of Lagrange function as used by Imorlica et al [16] to fuzzy set. FURIA is an equivalent of Repeated Incremental Pruning to Produce Error Reduction (RIPPER) with the main difference being that with RIPPER, the rules are crispy whereas in FURIA the rules are fuzzy. The rules take the form $r = \langle rA|rC \rangle$, consisting of a premise rA and a consequence rC . The premise is a conjunction of predicates (selectors) which take the form $(A_i = v_i)$ for nominal and $(A_i \mu v_i)$ for numerical attributes, where $\mu \in \{\leq, =, \geq\}$ and $v_i \in D_i$. The consequence part rC is a class assignment of the form $(class = \lambda)$, where $\lambda \in L$.

Whereas the RIPPER rule set takes on a range of 5 crisp numbers represented by 1-5 in the experiment, the FURIA takes on membership functions which are fuzzified from the crisp numbers as represented in Fig 3. The use of fuzzy numbers is meant to address the vagueness of human judgement [17].

As part of the efforts to address the vagueness in human reasoning when it comes to decision making, Ahmed and Kilic [17] proposed the use of centroid defuzzification with their results producing increased accuracy in weights. In the centroid defuzzification, a crisp number like 3 would have an upper bound of 4, the mid-point 3 and a lower bound of 2 resulting in the conversion of crisp number 3 to fuzzy number (2,3,4). Using the same concept, we converted all the crisp numbers to fuzzy numbers (Table 2) with age for example converting from 3 to (2, 3, 4) and source of income with crisp value 5 is converted to fuzzy value (4, 5, 6).

| Crisp number | Description | Fuzzy number |
|--------------|----------------------|--------------|
| 1 | Not important at all | (1, 1, 1) |
| 2 | Somewhat important | (1, 2, 3) |
| 3 | Important | (2, 3, 4) |
| 4 | Very important | (3, 4, 5) |
| 5 | Extremely important | (5, 5, 5) |

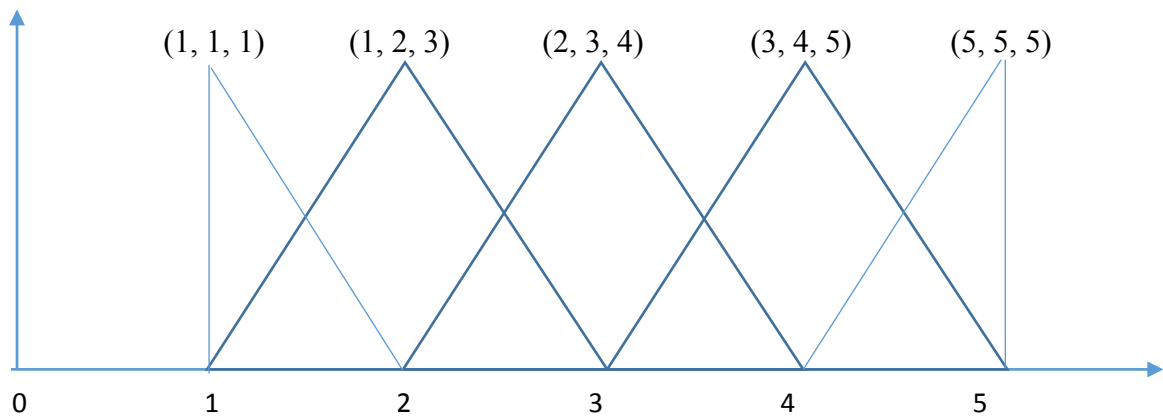


Figure 2: Fuzzification of crisp numbers

In a multi-step determination of the relative importance of the determinants, consistency index is used. The Fuzzy Analytical Hierarchical Process (FAHP) steps are as represented in Figure 3.

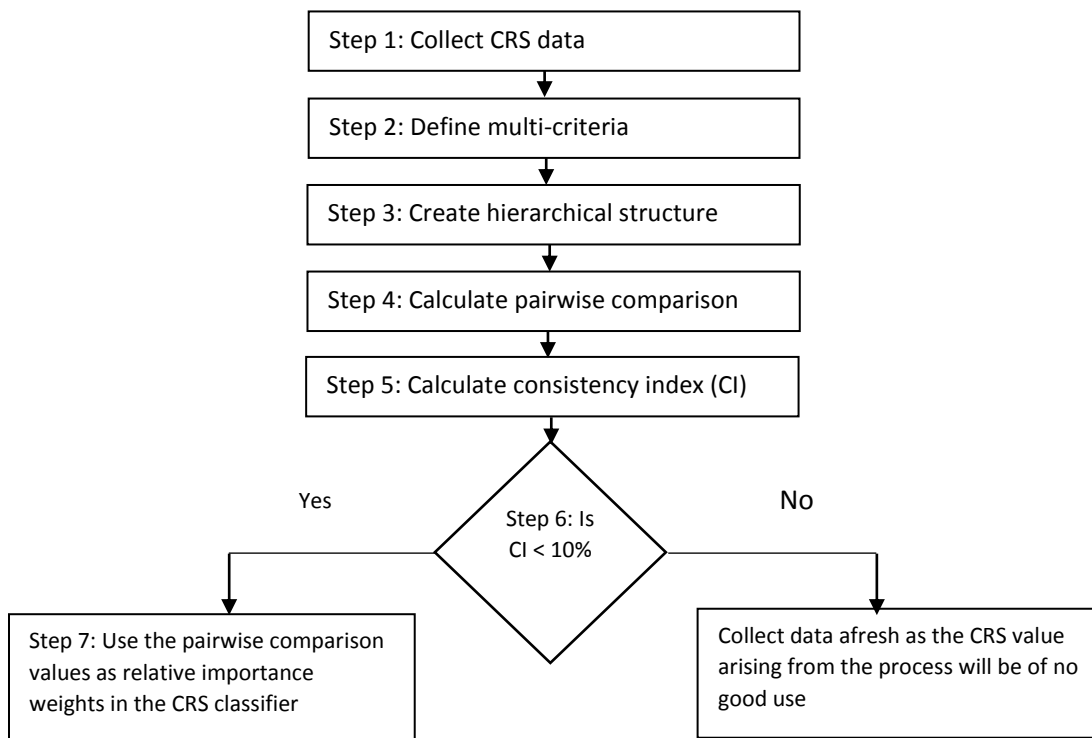


Figure 3: FAHP Process for CRS

Pairwise comparisons for the fuzzy numbers is done by getting the ratio of the fuzzy numbers, for example, getting pairwise comparison of age to source of income would be getting ratio of age/source of income which evaluates to

$$\text{pairwise comp} \left(\frac{\text{age}}{\text{source of income}} \right) = \left(\frac{2}{4}, \frac{3}{5}, \frac{4}{6} \right) = (0.5, 0.6, 0.67)$$

Working from the pairwise comparison of the criteria, λ_{\max} is calculated as the sum of the ratios between the Weighted sum value/ Criteria weight which in our case evaluates to **14.02** for FAHP. We then calculate the consistency index as proposed by Saaty [10] which is given by:

$$\text{Consistency index (CI)} = \frac{\lambda_{\max} - n}{n - 1}$$

$$\text{Consistency index (CI)} = \frac{14.02 - 14}{14 - 1} = 0.000836551323884622$$

Consistency ratio is evaluated as

$$\text{Consistency ratio} = \frac{\text{Consistency index}}{\text{Random index}}$$

Where the random index is taken from the Saaty random index table. For n = 14, the table returns random index of 1.57. So, in our case, the consistency ratio evaluates to

$$\text{Consistency ratio} = \frac{0.000836551323884622}{1.57} = 0.000529462863218115$$

The consistency ratio is less than the benchmark of 0.1 implying consistency in the comparisons [19]. It is important to note that AHP returned a consistency ratio of 0.44 which is higher than the 0.0005 for FAHP implying that fuzzy gives better results than crisp making it a better criteria weights for credit scoring process.

iii. Repeated game

Suppose T represent the total arms and N the total negatives contributing to regret, if the positives covered is given by p and the negatives covered by n, the success rate is given by p/T and the pruning rule is given by

$$(\text{Rule, PrunePos, PruneNeg}) = \frac{p - n}{p + n}$$

The game is repeated several times, each time growing and pruning the rules with each readjustment of the regret. Suppose a rule have k conditions of n possible conditions, pr be known by the message recipient (pr = k/n) and ||k|| be the number of bits needed to send integer k, the pruning function s evaluates to

$$s(n, k, pr) = k \log_2 \frac{1}{pr} = (n - k) \log_2 \frac{1}{1 - pr} + ||k|| \times 0.5$$

The regret bound is given by

$$OPT - \sum_{t=1}^T r_1(a_t) \leq \sigma f(K, T, B, d)$$

Where OPT is the maximum possible reward, K is the number of arms, d are the resources, B the loan headroom (budget), T are the time steps in the CRS and $r_i(a_t)$ are the actual realized reward.

6. Technology use case

Difficulties in credit scoring for smallholder farmers has always led to lenders shying away from lending to smallholder farmers. As a departure from index-based risk scoring, which works against the borrowers, this paper has proposed a CRS algorithm that can be modelled as a BwK problem using fuzzy sets. This gives the advantage of improving the regret in the fuzzified explore-exploit cycles. The use of fuzzy instead of crisp rule set gives the

advantage of making the computer generated CRS decision closer to human imprecise thinking.

The paper proposes that lenders should adopt the exploration-exploitation mechanism proposed here in their computer-based credit risk scoring. The agility and near-human reasoning provided by the fuzzy logic, combined with the optimal exploitation-exploration mechanism provided by the bandits with knapsacks mechanism has potential of reducing the error rate in risk scoring while providing cheap risk scoring tools. This has the ultimate chance of increasing lender confidence in risk scoring for smallholder farmers hence improving the financing of smallholder farmers.

7. Conclusions

The objective of the paper was to propose a mechanism that would improve the exploration-exploitation challenge in CRS for smallholder farmers. The paper proposed the use of BwK, a mechanism that has not been used before in CRS. Nested in the BwK is the fuzzy logic (FURIA) employing FAHP as a multi-criteria decision method. The experiment gives fairly low consistency ratios (**0.000529** in this case) for smallholder farmers CRS problem making it reasonable to use FAHP in the FURIA decision-making process. FURIA can then be used as the ϵ -greedy strategy in the BwK problem giving more confidence to lenders on the resultant credit scores. The paper confirms that the use of BwK using fuzzy has a potential of reducing the cost of credit risk scoring for smallholder farmers while at the same time providing near-human reasoning in the credit risk scoring process.

The paper assumed a stochastic linear function, which may not be true for a typical CRS problem. Further experimentations on adversarial functions which may produce better results is proposed. Lenders should actively engage technologists in incorporating BwK and fuzzy sets in their risk scoring methods for smallholder farmers and retail borrowers in general.

References

- [1] W. Chen, Y. Wang and Y. Yuan, "Combinatorial multi-armed bandit: General framework and applications," in *In International Conference on Machine Learning*, 2013.
- [2] S. Ontanón, "The combinatorial multi-armed bandit problem and its application to real-time strategy games," in *In Proceedings of the Ninth AAI Conference on Artificial Intelligence and Interactive Digital Entertainment*, 2013.
- [3] B. Otieno, F. Wabwoba and G. Musumba, "Towards Small-Scale Farmers Fair Credit Scoring Technique," in *2020 IST-Africa Conference (IST-Africa)*, IEEE, 2020..
- [4] D. Marron, "Lending by numbers: credit scoring and the constitution of risk within American consumer credit," *Economy and society*, vol. 36, no. 1, pp. 103-133, 2007.
- [5] B. Baesens, T. V. Gestel, S. Viaene, M. Stepanova, J. Suykens and J. Vanthienen, "Benchmarking state-of-the-art classification algorithms for credit scoring," *Journal of the operational research society*, vol. 54, no. 6, pp. 627-635, 2003.
- [6] S. Lessmann, B. Baesensb, H.-V. Seow and L. C. Thomas, "Benchmarking state-of-the-art classification algorithms for credit scoring," *Journal of the Operational Research Society*, 2017.
- [7] A. Badanidiyuru, R. Kleinberg and A. Slivkins, "Bandits with knapsacks," in *2013 IEEE 54th Annual Symposium on Foundations of Computer Science*, IEEE, 2013 .
- [8] T. a. S. S. Kesselheim, "Online Learning with Vector Costs and Bandits with Knapsacks," in *2020, PMLR, Conference on Learning Theory*.
- [9] A. J. L. a. A. S. Badanidiyuru, "Resourceful contextual bandits," in *Conference on Learning Theory*, 2014.
- [10] Y. Wind and T. L. Saaty, "Marketing applications of the analytic hierarchy process," *Management science*, vol. 26, no. 7, pp. 641-658, 1980.
- [11] D.-Y. Chang, "Applications of the extent analysis method on fuzzy AHP," *European journal of operational research*, vol. 95, no. 3, pp. 649-655, 1996.

- [12] Y.-M. Wang, Y. Luo and Z. Hua, "On the extent analysis method for fuzzy AHP and its applications," *European journal of operational research* , vol. 186, no. 2, pp. 735-747, 2008.
- [13] K.-J. Y. J. a. D.-Y. C. Zhu, "A discussion on extent analysis method and applications of fuzzy AHP," *European journal of operational research* , vol. 116, no. 2, pp. 450-456, 1999.
- [14] L. C. Leung and D. Cao, "On consistency and ranking of alternatives in fuzzy AHP," *European Journal of Operational Research* , vol. 124, no. 1, pp. 102-113, 2000.
- [15] G. Lugosi, M. Markakis and G. Neu, "On the Hardness of Learning from Censored Demand," *Available at SSRN 3509255* , 2019.
- [16] N. Immorlica, K. A. Sankararaman, R. Schapire and A. Slivkins, "Adversarial bandits with knapsacks," in *Annual Symposium on Foundations of Computer Science (FOCS)*, pp. 202-219. *IEEE*, 2019.
- [17] F. Ahmed and K. Kilic, "Modification to fuzzy extent analysis method and its performance analysis," in *International Conference on Industrial Engineering and Systems Management (IESM)*, *IEEE*, 2015.
- [18] K. Teknomo, Analytic hierarchy process (AHP) tutorial, *Revoledu. com* , 2006.
- [19] K. D. M. Wolfenson, "Coping with the food and agriculture challenge: smallholders' agenda," *Food and Agriculture Organization of the United Nations*, Rio, 2013.
- [20] A. M. Ali-Olubandwa, N. J. Kathuri, D. Odero-Wanga and W. A. Shivoga, "Challenges Facing Small Scale Maize Farmers ges Facing Small Scale Maize Farmers Agricultural Reform Era," *American Journal of Experimental Agriculture*, pp. 466-476, 2011.
- [21] R. W. Saaty, "The analytic hierarchy process—what it is and how it is used," *Mathematical modelling* 9.3-5 , pp. 161-176, 1987.