DEDAN KIMATHI UNIVERSITY OF TECHNOLOGY

UNIVERSITY EXAMINATIONS 2020/2021 ACADEMIC YEAR

FOURTH YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF TECHNOLOGY IN BUILDING CONSTRUCTION

TBD 2303: ENGINEERING MATHEMATICS I/

TBD 3103: ENGINEERING MATHEMATICS

DATE: 8TH DECEMBER 2021 **TIME: 8:30-10:30AM**

QUESTION ONE (30 MARKS)

a) If the function $f(x) = \begin{cases} \dfrac{16-x^2}{x-4} & x \neq 4 \\ P & x = 4 \end{cases}$

is continuous, find the value of P (3 Marks)

b) Evaluate

i) $\lim\limits_{x\to\infty} \dfrac{6-4x}{\sqrt{5+8x^2}}$ (5 marks)

ii) $\lim\limits_{x\to 0} \dfrac{e^x - e^{-x}}{\sin 3x}$ (3 marks)

c) Find the domain of the function and represent it on a graph.

$f(x, y) = \sqrt{9-3x^2-y^2}$ (5 marks)

d) Given that if $u = \cos\sqrt{x^2 + y^2}$ find $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 u}{\partial y^2}$ (5 marks)

e) Find the maclaulin series for the function $f(x) = e^x \sin x$ (5 marks)

f) Given the function $z = xy - 2x$ and that $x = 1$, $y = -3$, $\Delta x = -0.01$ *and* $\Delta y = 0.02$

Determine value of Δz *and* dz (4 marks)

QUESTION TWO (20 MARKS)

a) i) State the mean value theorem of differential calculus.

iii) Find the mean value of the function $f(x) = 2x^2 - 7x + 10$ on the interval [2, 5]

(5 marks)

b) Find the value of $\frac{\partial z}{\partial y}$, $\frac{\partial^2 z}{\partial x \partial y}$ given $z = \frac{\cos x^3 y}{x^3}$

Where z is a function defined by two variables x *and* y (5 marks)

c) Find the extreme values of the function $f(x, y) = x^2 + 5xy + 2y^2 = 0$ (5 marks)

d) Use Maclaurin's series approximation to express $\log_e(1-x)$ as a polynomial of the fourth degree. Use your result to evaluate $\log_e 0.8$ (5 marks)

QUESTION THREE (20 MARKS)

a) State Rolle's theorem

Hence justify Rolle's Theorem for the function $f(x) = x^3 - 3x^2 + x + 1$ on the interval $\left[1, 1+\sqrt{2}\right]$

(5 marks)

b) Find the value of $\frac{\partial^2 f}{\partial x \partial y}$ for the function $f(x, y) = y + x^{-2}y^2 + 4x^3 e^{-4y} - \ln\left(x^2 + y\right)$ (4 Marks)

c) A rectangular box, open at the top, is to have a volume of 64 cubic feet. What must be the Dimensions so that the total surface is a minimum? (6 marks)

d) By considering the paths along $x = 0$, $y = 0$ *and* $kx^2 = y$ show that the function $f(x.y) = \frac{x^2 - y}{x^2 + y}$ has no limit as $(x, y) \to (0, 0)$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Find the extrema of the function $f(x, y) = 4x - 2y$ subject to constraint $x^2 + y^2 = 1$ using the Lagrange's multiplier. (4 marks)

b) Find values of A and B for which the function is continuous for all values of x

$$f(x) = \begin{cases} Bx - 1 & x < -1 \\ Ax^2 - 2Bx + 1 & -1 \le x \le 1 \\ 3A & x > 1 \end{cases}$$ (5 marks)

c) Given that $f(x, y) = \sin xy + xe^y$ show that

$$\left.\frac{\partial f}{\partial x}\right|_{(0,2)} - \left.\frac{\partial f}{\partial y}\right|_{(3,0)} = e^2 - 4$$ (4 marks)

d) Use Taylors approximation to express $\sin\left(\frac{®}{3} + h\right)$ in ascending powers of h up to h^4

Taking $\sqrt{3} = 1.7321$ and 5.5^0 *as* 0.09599 radians, find the value of $\sin 54.5$ (7 marks)

QUESTION FIVE (20 MARKS)

a) State Rolle's theorem, hence verify Rolle's theorem for

$f(x) = 1 + \sin x$ in the interval $\left[-\frac{®}{2}, \frac{®}{2}\right]$ (4 marks)

b) Find Taylor series expansion of the integral at $x_0 = 1$

$$\int x^2 \ln x$$ (5 marks)

c) Determine the critical points of the function $f(x, y) = x^2 + 3xy + y^2$ (5 marks)

d) Use limits to find all the asymptotes

$$f(x) = \frac{x^3 - x}{x^2 - 6x + 5}$$ (6 marks)